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IDENTIFIERS	*School Mathematics Study Group

ABSTRACT

These masters are designed to be used in preparing transparencies for use with overhead projectors in conjunction with a School Mathematics Study Group publication for junior high school mathematics teachers. (MK)

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STUDIES IN MATHEMATICS

VOLUME XII

A BRIEF COURSE IN MATHEMATICS FOR JUNIOR HIGH SCHOOL TEACHERS

SE 029 957

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Financial support for the School Mathematics Study Group has been provided by the National Science Foundation.

PURPOSE

These masters are designed to be used in preparing completed projectuals (transparencies) for use with overhead projectors in conjunction with the School Mathematics Study Group publication, Studies in Mathematics, Volume XII, A Brief Course in Mathematics for Junior High School Teachers. This text develops the basic content necessary to understand and teach the material covered in the seventh grade SMSG text, Mathematics for Junior High School, Volume I.

Some of the projectuals ask questions and provide space for answers to be written following class discussion.

PREPARATION OF PROJECTUALS

For the proper use of these masters in preparing the individual sheets for the projectuals, consult the directions supplied with your own copying machine. The directions for mounting the sheets in each projectual are given in detail here.

To help in assembling, each master sheet is labeled in the top center. The first number indicates the chapter; the second number indicates the position of the projectual within the sequence for the chapter. The sheet labeled BASIC for each numbered projectual is to be permanently attached to the mount. The others are to be hinged on the side indicated, LEFT, RIGHT, or BOTTOM.

When assembling each projectual, be sure the registration marks on the individual sheets coincide before the sheets are attached to the mount. The small numbers in the upper right or left indicate the position of the sheet within the sequence of overlays for the particular projectual. When more than one overlay is used in a projectual, the order is usually obvious: (1) LEFT, (2) RIGHT, (3) BOTTOM. In a few projectuals each individual overlay needs to be removed before the next is used.

Topics for which masters are included are listed on the next page. In each case the number of sheets for the completed projectual is indicated.

TOPICS INCLUDED ON MASTERS

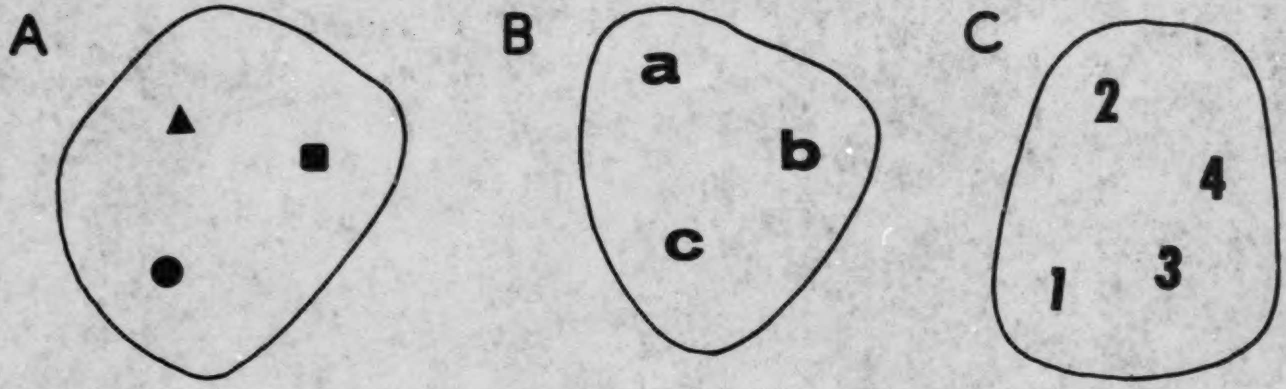
Chapter		Position in Chapter Sequence	Number of Sheets in Projectual	Topic
1	-	1	1	Sets
1	-	2	4	One-to-one correspondence
1	-	3	3	Equal sets
1	-	4	3	Equivalent sets
1	-	5	3	Subsets
1	-	6	3	Number of Subsets
1	-	7	3	Union and intersection
1	-	8	4	Solution sets for open sentences
1	-	9	4	Union
1	-	10	4	Intersection
1	-	11	4	Compound number sentences
2	-	1	4	Grouping
2	-	2	3	Counting using numerals in different bases
2	-	3	3	Place value
2	-	4	4	Conversions from one base to another
3	-	1	3	Base five tables
3	-	2	4	Addition base five
3	-	3	4	Multiplication base five
3	-	4	3	Base three tables
3	-	5	1	Ordering numbers expressed in different bases
3	-	6	2	Addition table base eight

Chapter		Position in Chapter Sequence	Number of Sheets in Projectual	Topic
4	-	1	1	Binary operation I
4	-	2	1	Binary operation II
4	-	3	2	Binary operation *
4	-	4	3	Commutative property
4	-	5	3	Associative property
4	-	6	3	Identity element
4	-	7	3	Inverse elements
4	-	8	3	Clock arithmetic
4	-	9	3	Distributive property
4	-	10	2	Operation ANTH
5	-	1	1	Properties of the whole numbers
5	-	2	3	Rational numbers
5	-	3	3	The set of integers
5	-	4	3	Integers
5	-	5	4	Adding integers
6	-	1	3	Addition of rational numbers I
6	-	2	3	Addition of rational numbers II
6	-	3	4	Multiplication with integers
7	-	1	2	Least common multiple
7	-	2	2	Greatest common factor
7	-	3	1	Prime factorization
7	-	4	4	Sieve of Erathosthenes
8	-	1	2	Expanded notation

Chapter		Position in Chapter Sequence	Number of Sheets in Projectual	Topic
9	-	1	2	Repeating decimals (1)
9	-	2	3	Repeating decimals (2)
9	-	3	3	Locating a rational and an irrational between two rationals
9	-	4	4	Locating $\sqrt{2}$
9	-	5	4	Locating π on the number line
10	-	1	4	Sketching
10	-	2	1	Solids
10	-	3	3	Property 1: Through any two points in space there is exactly one line
10	-	4	1	Lines determined by points
10	-	5	2	Property 2: If a line contains two different points of a plane
10	-	6	4	Property 3: Any three points not in the same straight line are in exactly one plane
10	-	7	3	Two lines in space
10	-	8	4	A line and a plane in space
10	-	9	1	Property 4: If the intersection of two different planes is not empty then the intersection is a line
10	-	10	1	Planes in space
11	-	1	4	Interior of angle ABC
11	-	2	4	Interior of triangle ABC
11	-	3	2	Lines, angles, rays and segments
11	-	4	1	Diagonals of polygons
11	-	5	2	Cylindrical surfaces
11	-	6	2	Conical surfaces

Chapter		Position in Chapter Sequence	Number of Sheets in Projectual	Topic
11	-	7	1	V - E + F = 2 (1)
11	-	8	1	V - E + F = 2 (2)
11	-	9	2	V - E + F = 2 (3)
11	-	10	4	V - E + F = 2 (4)
12	-	1	4	One-to-one correspondence
12	-	2	4	Precision-linear measurement
12	-	3	4	Angles of a triangle--180°
13	-	1	4	Area
13	-	2	4	Volume
13	-	3	3	Volume - $\frac{1}{2}$ inch precision

A SET IS A COLLECTION OF OBJECTS


A
B
a
b
c
C
2
4
1
3

SETS MAY BE NAMED BY LISTING THEIR ELEMENTS

A = { ▲, ●, ■,}

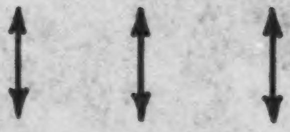
B = { a, b, c,}

C = { ▨, ▨, ▨, ▨,}

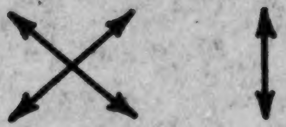
ONE-TO-ONE CORRESPONDENCE IS A PAIRING OF EACH ELEMENT OF ONE SET WITH AN ELEMENT OF A SECOND SET AND CONVERSELY.

A { ▲, ●, ■ }

B { a, b, c }

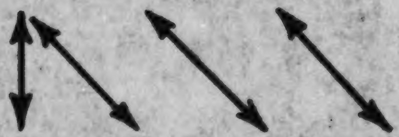


THIS IS A ONE-TO-ONE CORRESPONDENCE BETWEEN SET A AND SET B.



THIS IS ANOTHER ONE-TO-ONE CORRESPONDENCE BETWEEN SET A AND SET B .

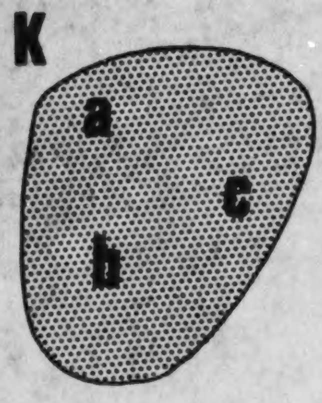
1 - 2 Bottom



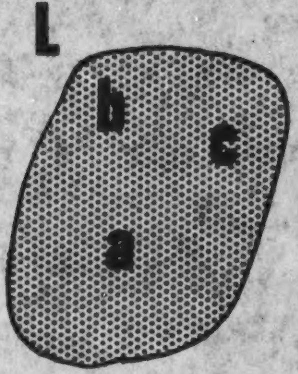
C $\{\ 1,\ 2,\ 3,\ 4\ \}$

IS THIS A ONE-TO-ONE CORRESPONDENCE BETWEEN SET B AND SET C ?

EQUAL SETS ARE SETS WHICH CONTAIN EXACTLY THE SAME ELEMENTS.



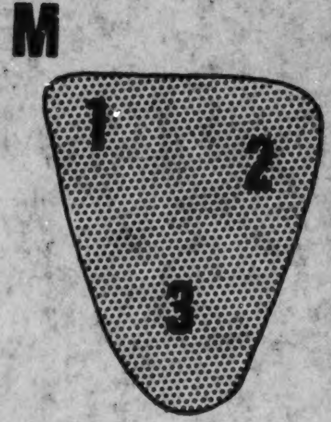
K= $\{$ **a, b, c** $\}$



$L = \{ b, a, c \}$

SET **K** EQUALS SET **L**

$K = L$

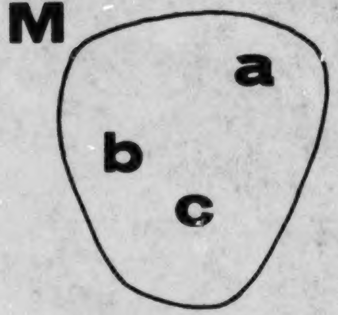


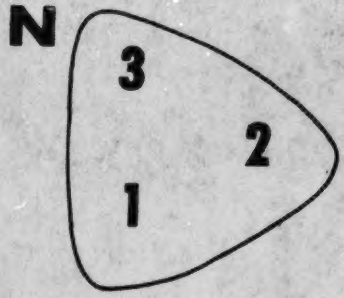
$M = \{1, 2, 3\}$

DOES SET **L** EQUAL SET **M** ?

DOES SET **K** EQUAL SET **M** ?

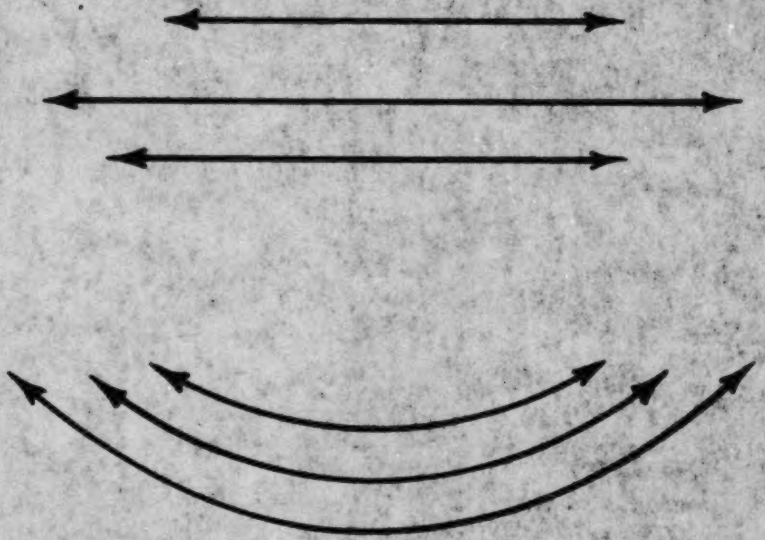
EQUIVALENT SETS ARE SETS WHICH MAY BE PLACED IN ONE-TO-ONE CORRESPONDENCE.



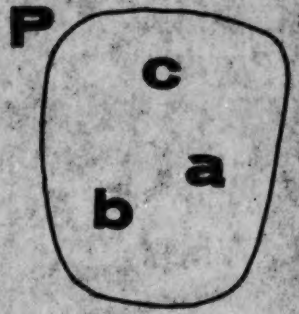


$M = \{a, b, c\}$ $N = \{1, 2, 3\}$

18



SETS **M** AND **N** ARE EQUIVALENT SETS.



P = {b, c, a}

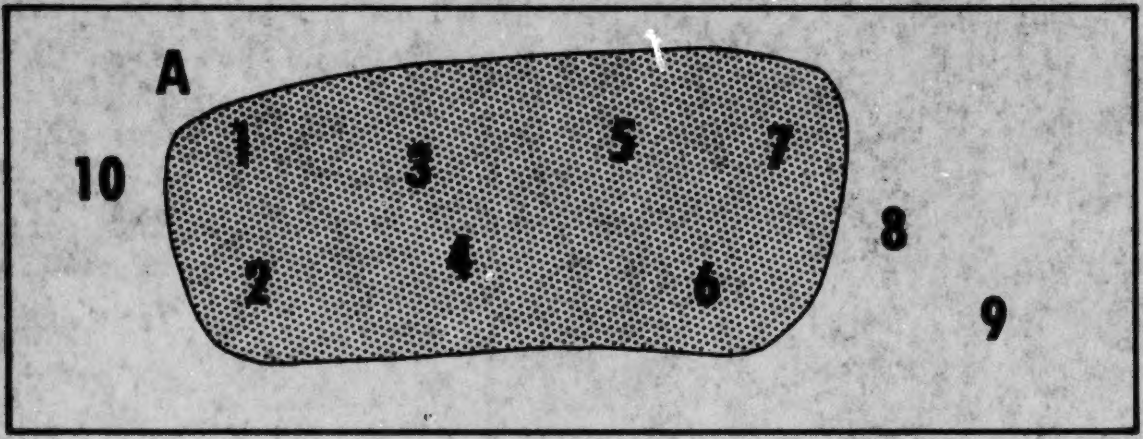
ARE SETS **M** AND **P** EQUIVALENT SETS ?

ARE SETS **N** AND **P** EQUIVALENT SETS ?

ARE SETS **M** AND **P** EQUAL SETS ?

ARE ALL EQUAL SETS EQUIVALENT SETS?

SUBSETS


A
10
1
2
3
4
5
6
7
8
9

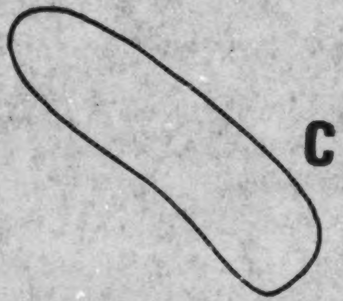
A = $\{1, 2, 3, 4, 5, 6, 7\}$

B $\{2,4,6\}$

EVERY ELEMENT OF **B** IS AN ELEMENT OF **A**.

B IS A SUBSET OF **A**.

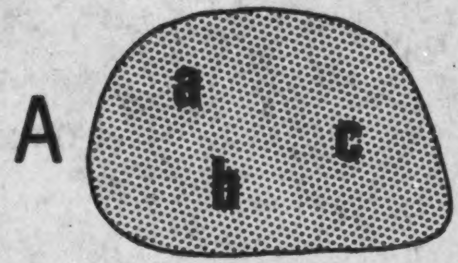
$$B \subset A$$



C $\{7, 8, 9\}$

IS **C** A SUBSET OF **A** ?

HOW MANY SUBSETS ?



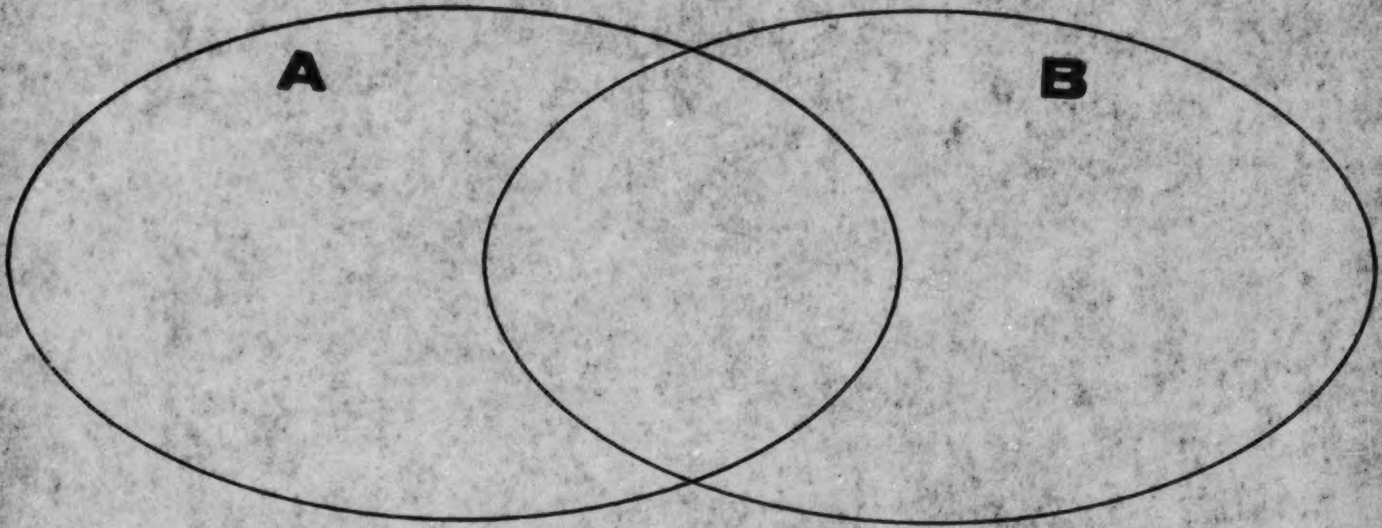
$A = \{a, b, c\}$

$$\begin{array}{llll}
\{a, b, c\} & \{a, b\} & \{a\} & \emptyset \\
 & \{a, c\} & \{b\} & \\
 & \{b, c\} & \{c\} &
\end{array}$$

$1 + 3 + 3 + 1 = 8$ SUBSETS

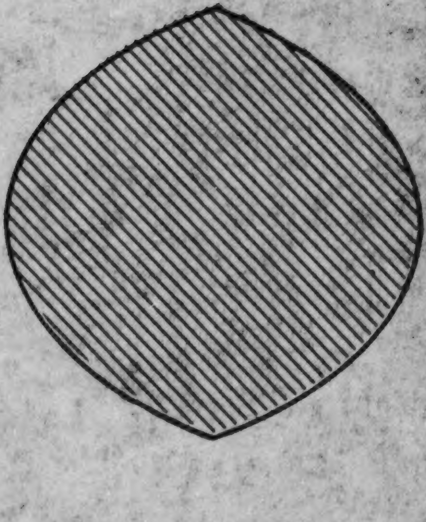
$$2^3 = 8$$

IN GENERAL, A SET OF n ELEMENTS HAS 2^n SUBSETS.


A
B

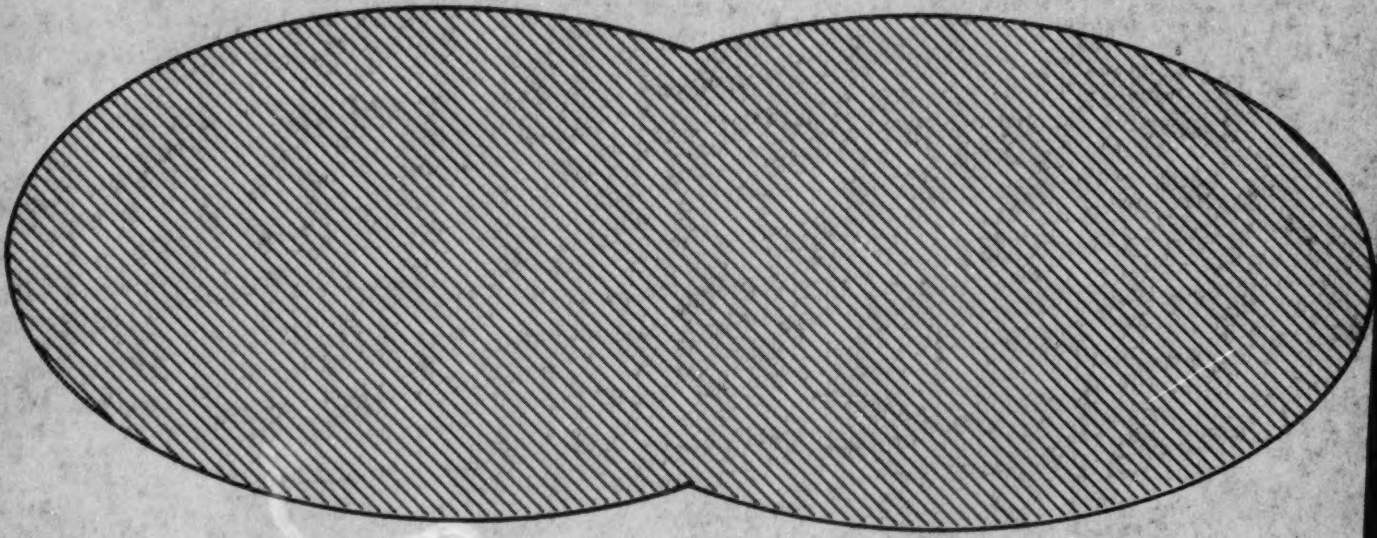
1 - 7 left

$A \cap B$



1 - 7 Right

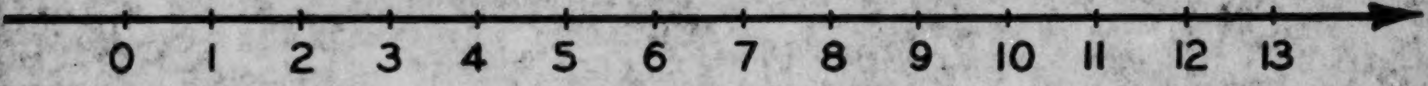
A U B



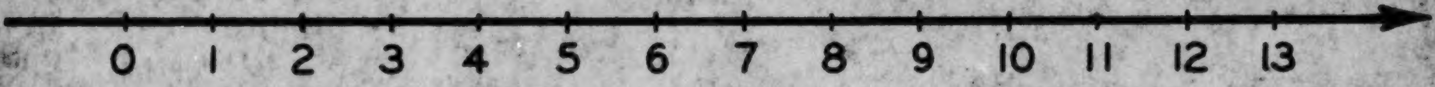
SOLUTION SETS FOR OPEN SENTENCES

REPLACEMENT SET:

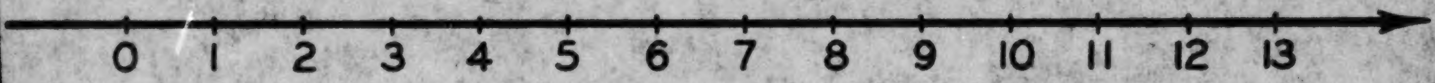
$x \leq 3$


0
1
2
3
4
5
6
7
8
9
10
11
12
13

$2 + x = 5$


0
1
2
3
4
5
6
7
8
9
10
11
12
13

$x + 2 > 6$


0
1
2
3
4
5
6
7
8
9
10
11
12
13

$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$\{0, 1, 2, 3\}$

● ● ● ●

$\{3\}$

●

$\{5, 6, 7, 8, 9, 10\}$

● ● ● ● ● ●

1 - 8 Right

$\{2, 4, 6, 8, 10\}$

$\{2\}$

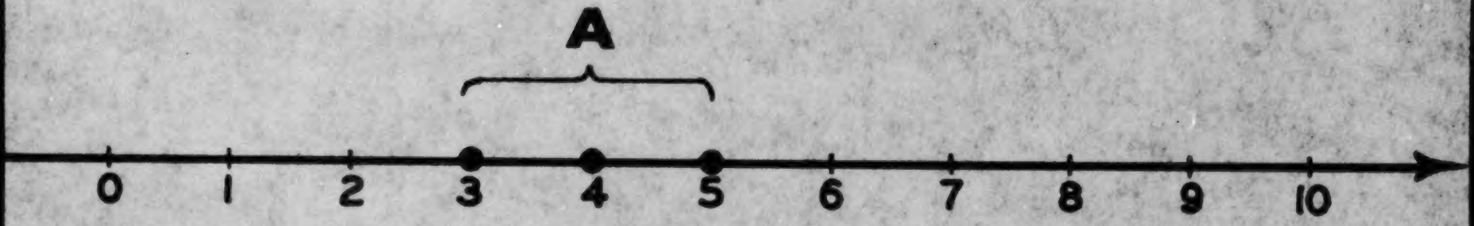
$\emptyset$

$\{6, 8, 10\}$

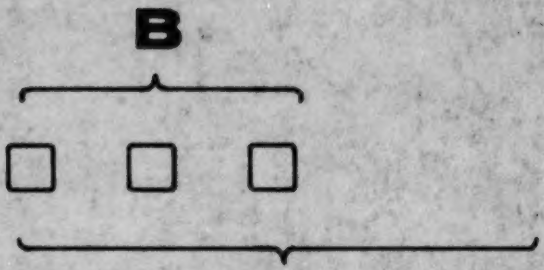
$\{0, 1, 2, 3 \ldots\}$

UNION

A = $\{3, 4, 5\}$


A
0
1
2
3
4
5
6
7
8
9
10

$\mathbf{B} = \{1, 2, 3\}$



$\mathbf{A \cup B} = \{1, 2, 3, 4, 5\}$

$$C = \{2, 4, 6\}$$

$$\underbrace{\square \qquad\qquad \square \qquad\qquad \square}_{A \cup C = \{2,3,4,5,6\}}$$

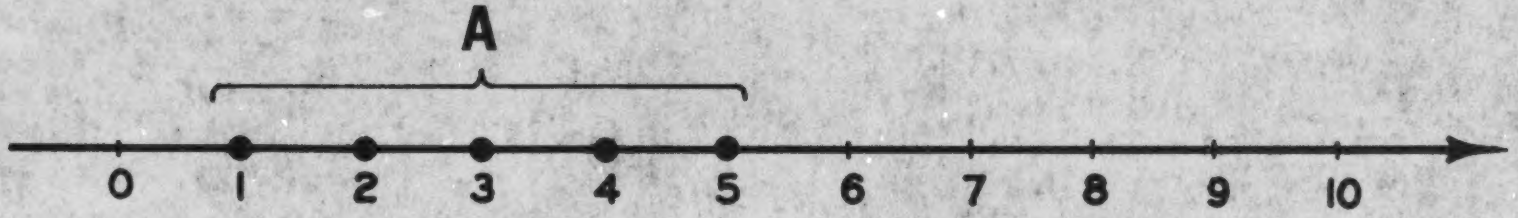
D = {9, 10} **E** = {1, 3, 5, 7, 9}

AUD =

AUE =

INTERSECTION

$A = \{1, 2, 3, 4, 5,\}$


A
0
1
2
3
4
5
6
7
8
9
10

1 - 10 Left

$$B = \{4, 5, 6, 7, 8\}$$

$$\overbrace{\underbrace{\square \quad \square}_{A \cap B = \{4, 5\}} \quad \square \quad \square \quad \square}^{B}$$

$$C = \{1, 2\}$$

$$\underbrace{\overbrace{\square \qquad \square}^{C}}$$

$$A \cap C = \{1, 2\}$$

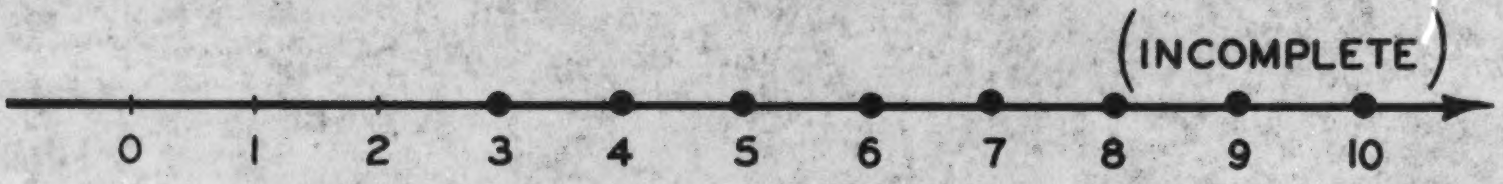
$D = \{7, 9\}$ $E = \{2, 4, 6, 8, 10\}$

$A \cap D =$

$A \cap E =$

COMPOUND NUMBER SENTENCES

REPLACEMENT SET: $\{0, 1, 2, 3, \ldots\}$ (WHOLE NUMBERS)


(INCOMPLETE)
0
1
2
3
4
5
6
7
8
9
10

NUMBER SENTENCE	SOLUTION SET
$X \geq 3$	$\{3, 4, 5, 6, \ldots\}$

$\square$ $\square$ $\square$ $\square$ $\square$ $\square$ $\square$

$X < 7$	$\{0, 1, 2, 3, 4, 5, 6\}$
$X \geq 3$ AND $X < 7$	$\{3, 4, 5, 6\}$

☐ ☐

$X \leq 1$	$\{0, 1\}$
$X \geq 3$ OR $X \leq 1$	$\{0, 1, 3, 4, 5, 6, \ldots\}$

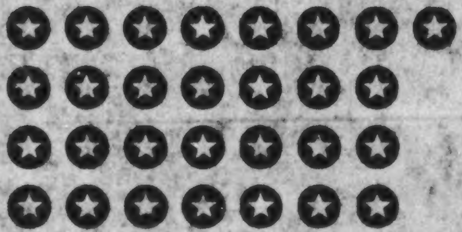
1 - 11 Bottom

$X \geq 3$ OR $X < 7$

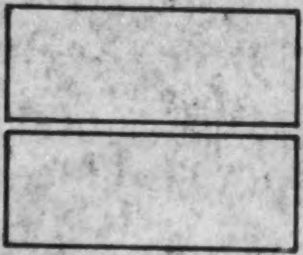
$X \geq 3$ AND $X \leq 1$

$X \geq 3$ AND $X > 7$

GROUPING



2 - 1 Left

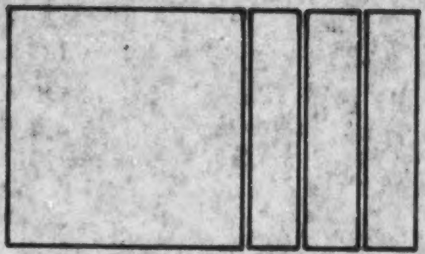


by TENS

2 TENS

9 ONES

29_{ten}



by FOURS

1 four x four

3 four

1 one

131_{four}

2 - 1 Bottom

by SEVEN

▨ sevens

▨ ones

▨ seven

COUNTING

BASE TEN 1 2 3 4 5 6 7 8 9 10

BASE TWO 10

BASE THREE 10

BASE FOUR 10

BASE FIVE 10

1		11	100	101	110	111	1000	1001	1010
1	2		11	12	20	21	22	100	101
1	2	3		11	12	13	20	21	22
▨	▨	▨	▨		▨	▨	▨	▨	▨

PLACE VALUE

	B^5	B^4	B^3	B^2	B^1	B^0
BASE B	B·B·B·B·B	B·B·B·B	B·B·B	B·B	B	1

2 - 3 Left

BASE TWO					2	
BASE THREE					3	
BASE FOUR					4	
BASE FIVE					5	

2 - 3 Right

32	16	8	4		1
243	81	27	9		1
1024	256	64	16		1
3125	625	125	25		1

BASE

BASE

CONVERSIONS

FROM BASE **four** TO BASE **ten**.

$$\square_{\text{FOUR}} = (\square \times 4^3) + (\square \times 4^2) + (\square \times 4^1) + (\square \times 4^0)$$

$$= (\square \times 64) + (\square \times 16) + (\square \times 4) + (\square \times 1)$$

$$= \square + \square + \square + \square$$

$$= \square_{\text{TEN}}$$

FROM BASE **ten** TO BASE **four**.

$$\square_{\text{TEN}} = (\square \times 64) + (\square \times 16) + (\square \times 4) + (\square \times 1)$$

$$= \square_{\text{FOUR}}$$

2 - 4 Left

1231	1	2	3	1
	1	2	3	1
	64	32	12	1
	109			
235	3	2	2	3
	3223			

2 - 4 Right

2103	2	1	0	3
	2	1	0	3
	128	16	0	3
	147			
188	2	3	3	0
	2330			

3022

209

63

BASE FIVE

ADDITION

+	0	1	2	3	4
0					
1					
2					
3					
4					

MULTIPLICATION

X	0	1	2	3	4
0					
1					
2					
3					
4					

3 - 1 Left

0	1	2	3	4
1	2	3	4	10
2	3	4	10	11
3	4	10	11	12
4	10	11	12	13

3 - 1 Right

0	0	0	0	0
0	1	2	3	4
0	2	4	11	14
0	3	11	14	22
0	4	13	22	31

ADDITION BASE FIVE

$$\begin{array}{r} 243_{\text{FIVE}} \\ +\underline{124}_{\text{FIVE}} \end{array}$$

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	10
2	2	3	4	10	11
3	3	4	10	11	12
4	4	10	11	12	13

12 ($3_{\text{FIVE}} + 4_{\text{FIVE}}$)

$$110(40_{\text{FIVE}} + 20_{\text{FIVE}})$$

$$\underline{\;300\;}\left(200_{\text{FIVE}} + 100_{\text{FIVE}}\right)$$

$$422_{\text{FIVE}}$$

MULTIPLICATION BASE FIVE

$$\begin{array}{r} 324_{\text{FIVE}} \\ \underline{\times\ 3_{\text{FIVE}}} \end{array}$$

×	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	11	13
3	0	3	11	14	22
4	0	4	13	22	31

$22(3_{\text{FIVE}} \times 4_{\text{FIVE}})$

$110\ (3_{\text{FIVE}} \times 20_{\text{FIVE}})$

72-A

$$\begin{array}{l}\underline{1400}\ (3_{\text{FIVE}} \times 300_{\text{FIVE}}) \\ 2032_{\text{FIVE}}\end{array}$$

BASE THREE

ADDITION

+	0	1	2
0			
1			
2			

MULTIPLICATION

X	0	1	2
0			
1			
2			

0	1	2
1	2	10
2	10	11

3 - 4 Right

0	0	0
0	1	2
0	2	11

COMPLETE WITH =, <, OR >.

40_{SIX} ____ 100_{FIVE}

1111_{TEN} ____ 15_{TEN}

202_{SEVEN} ____ 100_{TEN}

200_{EIGHT} ____ 231_{SEVEN}

112_{THREE} ____ 1010_{TWO}

$\text{TO}_{\text{TWELVE}}$ ____ 1100_{FIVE}

1120_{FOUR} ____ 143_{SEVEN}

11111_{TWO} ____ 37_{EIGHT}

10_{SIXTY} ____ 10_{THIRTY}

1010_{THREE} ____ 162_{SEVEN}

+	0	1	2	3	4	5	6	7
0								
1								
2								
3								
4								
5								
6								
7								

3 - 6 Left

0	1	2	3	4	5	6	7
1	2	3	4	5	6	7	10
2	3	4	5	6	7	10	11
3	4	5	6	7	10	11	12
4	5	6	7	10	11	12	13
5	6	7	10	11	12	13	14
6	7	10	11	12	13	14	15
7	10	11	12	13	14	15	16

BINARY OPERATION I

IF $2 \circ 5 = 6$

$3 \circ 3 = 5$

$4 \circ 7 = 10$

$1 \circ 2 = 2$ THEN $a \circ b$ ______

IF $3 \triangle 5 = 5$

$7 \triangle 1 = 7$

$6 \triangle 9 = 9$

$8 \triangle 8 = 8$ THEN $a \triangle b$ ______

BINARY OPERATION II

IF $3 \sim 4 = 10$

$1 \sim 3 = 5$

$3 \sim 0 = 6$

$5 \sim 1 = 11$ THEN $\mathbf{a} \sim \mathbf{b} =$ [redacted]

IF $3 \sqcup 4 = 5$

$1 \sqcup 2 = 9$

$2 \sqcup 8 = 2$

$7 \sqcup 5 = 0$ THEN $\mathbf{a} \sqcup \mathbf{b} =$ [redacted]

BINARY OPERATIONS *

*	A	B	C	D	E
A	D	E	A	B	C
B	E	A	B	C	D
C	A	B	C	D	E
D	B	C	D	E	A
E	C	D	E	A	B

$B * C =$ __________

$C * B =$ __________

$A * A =$ __________

$E * D =$ __________

$E * E =$ __________

$(A * B) * C =$ __________

$A * (B * C) =$ __________

COMMUTATIVE PROPERTY

~	△	□	⊙	╲
△	△	□	⊙	╲
□	□	⊙	╲	△
⊙	⊙	╲	△	□
╲	╲	△	□	⊙

4 - 4 Left

□ ∼ ⊙ = ______

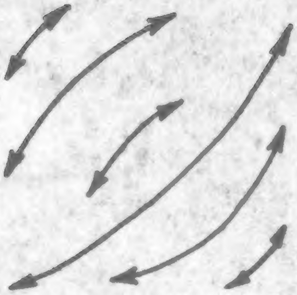
⊙ ∼ □ = ______

╲ ∼ □ = ______

□ ∼ ╲ = ______

△ ∼ ⊙ = ______

⊙ ∼ △ = ______



AN OPERATION $\sim$ DEFINED ON A SET **M** IS SAID TO BE **COMMUTATIVE** IF FOR ANY ELEMENT **X** AND **Y** OF **M**:

$$X \sim Y = Y \sim X$$

ASSOCIATIVE PROPERTY

∼	△	□	⊙	╲
△	△	□	⊙	╲
□	□	⊙	╲	△
⊙	⊙	╲	△	□
╲	╲	△	□	⊙

$(\backslash \sim \odot) \sim \square =$ ________

$\backslash \sim (\odot \sim \square) =$ ________

$(\square \sim \backslash) \sim \odot =$ ________

$\square \sim (\backslash \sim \odot) =$ ________

AN OPERATION $\sim$ *DEFINED ON A SET* **M** *IS SAID TO BE* **ASSOCIATIVE** *IF FOR ANY ELEMENTS X, Y, AND Z OF* **M**:

$$(X \sim Y) \sim Z = X \sim (Y \sim Z)$$

IDENTITY ELEMENT

∼	△	□	⊙	╲
△	△	□	⊙	╲
□	□	⊙	╲	△
⊙	⊙	╲	△	□
╲	╲	△	□	⊙

4 - 6 Left

□ ∼ △ = □

⊙ ∼ △ = ⊙

╲ ∼ △ = ╲

△ ∼ □ = □

△ ∼ ⊙ = ⊙

△ ∼ ╲ = ╲

△ ∼ △ = △

AN ELEMENT △ IS SAID TO BE AN **IDENTITY ELEMENT** *FOR THE OPERATION DEFINED ON SET* **M** *IF FOR EACH X OF* **M**:

$$X \sim \triangle = \triangle \sim X = X$$

INVERSE ELEMENTS

∼	△	□	⊙	╲
△	△	□	⊙	╲
□	□	⊙	╲	△
⊙	⊙	╲	△	□
╲	╲	△	□	⊙

4 - 7 Left

□ ∼ \ = △

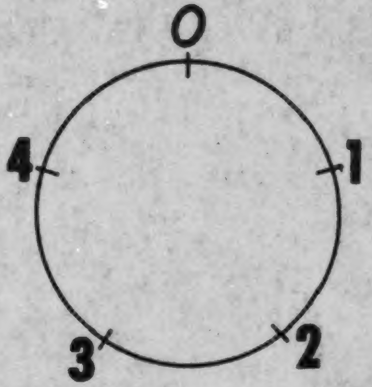
\ ∼ □ = △

⊙ ∼ ⊙ = △

△ ∼ △ = △

TWO ELEMENTS X AND Y OF THE SET **M** ARE SAID TO BE **INVERSES** OF EACH OTHER UNDER A BINARY OPERATION $\sim$ IF $X \sim Y = Y \sim X = \triangle$ WHERE $\triangle$ IS THE IDENTITY ELEMENT OF **M**.

CLOCK ARITHMETIC



ADDITION MOD 5

+	0	1	2	3	4
0					
1					
2					
3					
4					

MULTIPLICATION MOD 5

X	0	1	2	3	4
0					
1					
2					
3					
4					

4 - 8 Left

0	1	2	3	4
1	2	3	4	0
2	3	4	0	1
3	4	0	1	2
4	0	1	2	3

4 - 8 Right

0	0	0	0	0
0	1	2	3	4
0	2	4	1	3
0	3	1	4	2
0	4	3	2	1

DISTRIBUTIVE PROPERTY

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

X	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

$3 \times (4+2) = 3 \times \_\_\_$

$= \_\_\_$

$(3 \times 4) + (3 \times 2) = \_\_\_ + \_\_\_$

$= \_\_\_$

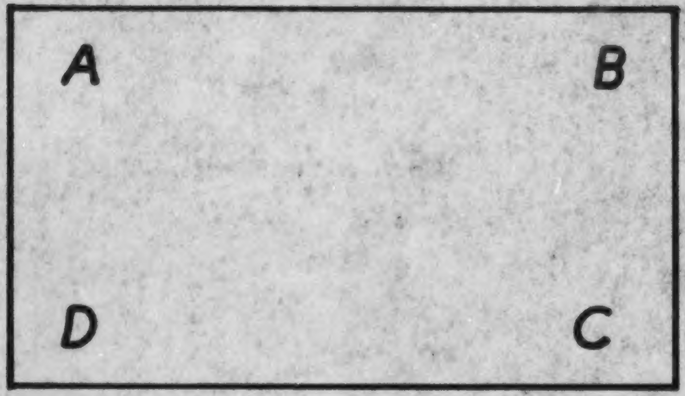
THE OPERATION X **DISTRIBUTES** *OVER THE OPERATION + IF* $A \times (B+C) = (A \times B) + (A \times C)$ *FOR ALL ELEMENTS A, B, AND C OF* **M.**

ANTH	I	V	H	R
I				
V				
H				
R				

4 - 10 Left

I V H R
V I R H
H R I V
R H V I

4 - 10 (Do not hinge - Use with 4-10 (1) in space indicated)



PROPERTIES OF THE WHOLE NUMBERS

(FOR WHOLE NUMBERS A, B, AND C)

COMMUTATIVE	$A + B = B + A$ $A \cdot B = B \cdot A$
ASSOCIATIVE	$(A + B) + C = A + (B + C)$ $(A \cdot B) \cdot C = A \cdot (B \cdot C)$
DISTRIBUTIVE	$A \cdot (B + C) = A \cdot B + A \cdot C$
ADDITIVE IDENTITY	$0 + A = A + 0 = A$
MULTIPLICATIVE IDENTITY	$1 \cdot A = A \cdot 1 = A$

RATIONAL NUMBERS

EQUATIONS	SOLUTIONS
$2x = 7$	
$9x = 4$	
$16x = 16$	

5 - 2 Left

7/2

4/9

16/16

THE SET OF SOLUTIONS TO ALL EQUATIONS OF THE FORM $BX = A$, WHERE A AND B ARE COUNTING NUMBERS, IS THE SET OF POSITIVE RATIONAL NUMBERS.

THE SET OF INTEGERS

1, 2, 3, . . .

THE COUNTING NUMBERS

0,

THE NUMBER ZERO

$$\left\{\underbrace{\ldots, 3,\ 2,\ 1,}_{\text{NEGATIVE INTEGERS}}\ \underbrace{\qquad}_{\text{ZERO}}\ \underbrace{\qquad\qquad\qquad\qquad}_{\text{POSITIVE INTEGERS}}\right\}$$

THE **OPPOSITES** OF THE COUNTING NUMBERS

INTEGERS

EQUATIONS	SOLUTIONS
$2 + x = 7$	
$9 + x = 4$	
$16 + x = 16$	

5 - 4 Left

5

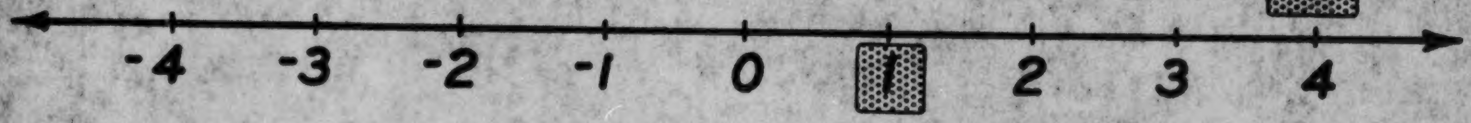
-5

0

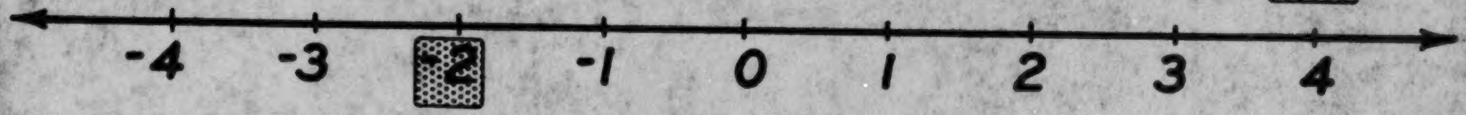
THE SET OF SOLUTIONS TO ALL EQUATIONS OF THE FORM $E + X = F$, WHERE E AND F ARE COUNTING NUMBERS, IS THE SET OF INTEGERS.

ADDING INTEGERS

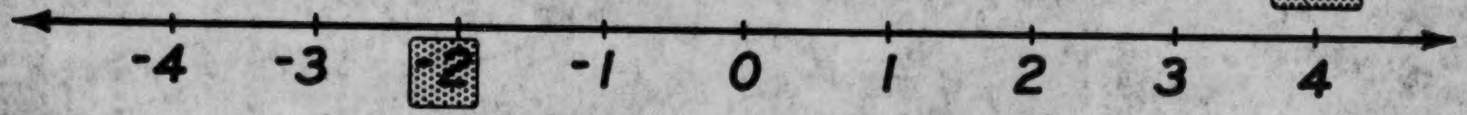
$1 + {}^{-}3 = \square$



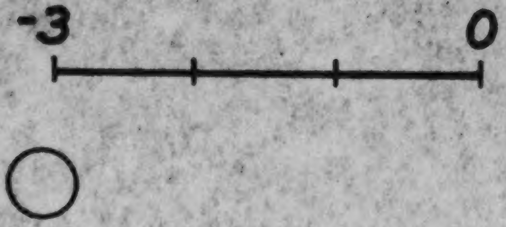
$^{-}2 + {}^{-}1 = \square$



$^{-}2 + {}^{-}2 = \square$



5 - 5 Left



-2

119-A

5 - 5 Right

0

ADDITION OF RATIONAL NUMBERS:

$$\frac{2}{3} + \frac{5}{8}$$

$\frac{2}{3}$ IS THE SOLUTION TO $3X = 2$

$$3 \cdot \frac{2}{3} = 2$$

$$8 \cdot \left(3 \cdot \frac{2}{3}\right) = 8 \cdot 2 = 16$$

$$(8 \cdot 3) \cdot \frac{2}{3} = 16$$

$\frac{5}{8}$ IS THE SOLUTION TO $8X = 5$

$$8 \cdot \frac{5}{8} = 5$$

$$3 \cdot \left(8 \cdot \frac{5}{8}\right) = 3 \cdot 5 = 15$$

$$(3 \cdot 8) \cdot \frac{5}{8} = 15$$

$(8 \cdot 3) \cdot \frac{2}{3} + (3 \cdot 8) \cdot \frac{5}{8} = 16 + 15$	ADDING EQUATIONS
$24 \cdot \frac{2}{3} + 24 \cdot \frac{5}{8} = 31$	MULTIPLICATION AND ADDITION OF WHOLE NUMBERS
$24 \left(\frac{2}{3} + \frac{5}{8} \right) = 31$	DISTRIBUTIVE PROPERTY

BUT $24 \cdot \frac{31}{24} = 31$

THEREFORE $\frac{2}{3} + \frac{5}{8} = \frac{31}{24}$

ADDITION OF RATIONAL NUMBERS: $\frac{2}{3} + \frac{5}{8}$

$\frac{2}{3} + \frac{5}{8} = \frac{2}{3} \cdot 1 + \frac{5}{8} \cdot 1$ MULTIPLICATION PROPERTY OF 1

$= \frac{2}{3} \cdot \frac{8}{8} + \frac{5}{8} \cdot \frac{3}{3}$ RENAMING 1

$= \frac{2 \cdot 8}{3 \cdot 8} + \frac{5 \cdot 3}{8 \cdot 3}$

DEFINITION OF MULTIPLICATION

$= \frac{16}{24} + \frac{15}{24}$ MULTIPLICATION OF COUNTING NUMBERS

$= \frac{1}{24}(16 + 15)$

6 - 2 Right

	DISTRIBUTIVE PROPERTY
$= \frac{1}{24}(31)$	ADDITION OF COUNTING NUMBERS
$= \frac{31}{24}$	DEFINITION OF MULTIPLICATION

MULTIPLICATION WITH INTEGERS

$7+(-7)=0$	ADDITIVE INVERSE
$4\,[7+(-7)]=4\cdot 0=0$	MULTIPLICATION BY 4
$4\cdot 7+4\cdot(-7)=0$	DISTRIBUTIVE PROPERTY

BUT $4 \cdot 7 + -(4 \cdot 7) = 0$ ADDITIVE INVERSE

THEREFORE $\boxed{4 \cdot (-7) = -(4 \cdot 7) = -28}$

$-4[7+(^{-}7)] = (^{-}4) \cdot 0 = 0$ MULTIPLICATION BY -4

$(^{-}4) \cdot 7 + (^{-}4) \cdot (^{-}7) = -(4 \cdot 7) + (^{-}4) \cdot (^{-}7) = 0$ DISTRIBUTIVE PROPERTY

BUT $-(4\cdot 7)+(4\cdot 7)=0$ ADDITIVE INVERSE

THERefore $\boxed{(^-4)\cdot(^-7)=(4\cdot 7)=28}$

LEAST COMMON MULTIPLE

MULTIPLES OF 20 — $C: \{20, 40, 60, 80, 100, 120\ldots\}$

MULTIPLES OF 30 — $D: \{30, 60, 90, 120, 150, \ldots\}$

MULTIPLES COMMON TO 20 AND 30

$C \cap D$: $\{60, 120, 180, \ldots\}$

LEAST COMMON MULTIPLE

l.c.m = 60

GREATEST COMMON FACTOR

FACTORS OF 20-A: {1,2,4,5,10,20}

FACTORS OF 30-B: {1,2,3,5,6,10,15,30}

FACTORS COMMON TO 20 AND 30-

$A \cap B$: {1,2,5,10}

GREATEST COMMON FACTOR

g. c. f. = 10

PRIME FACTORIZATION

PRIME FACTORIZATION OF 20: $2 \cdot 2 \cdot 5$

PRIME FACTORIZATION OF 30: $2 \cdot 3 \cdot 5$

g.c.f. = $2 \cdot 5 = 10$

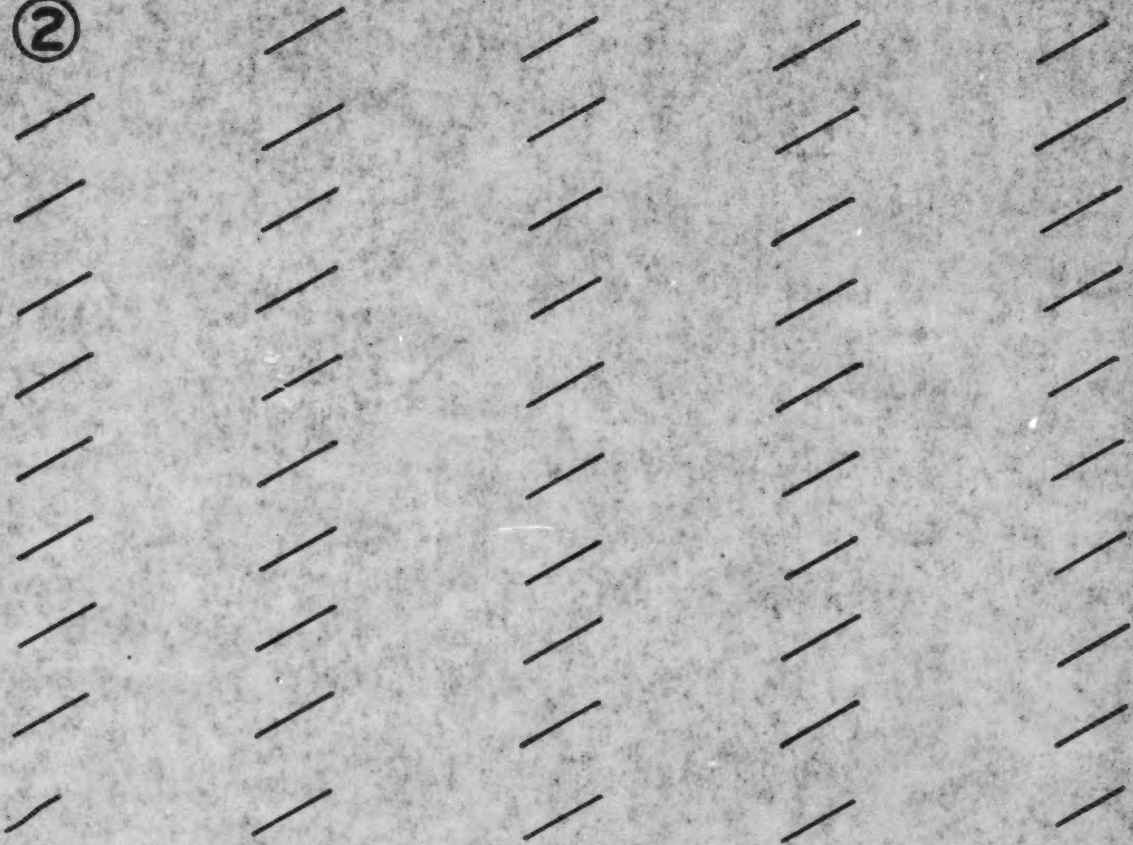
l.c.m. = $2 \cdot 2 \cdot 3 \cdot 5 = 60$

SIEVE OF ERATOSTHENES

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

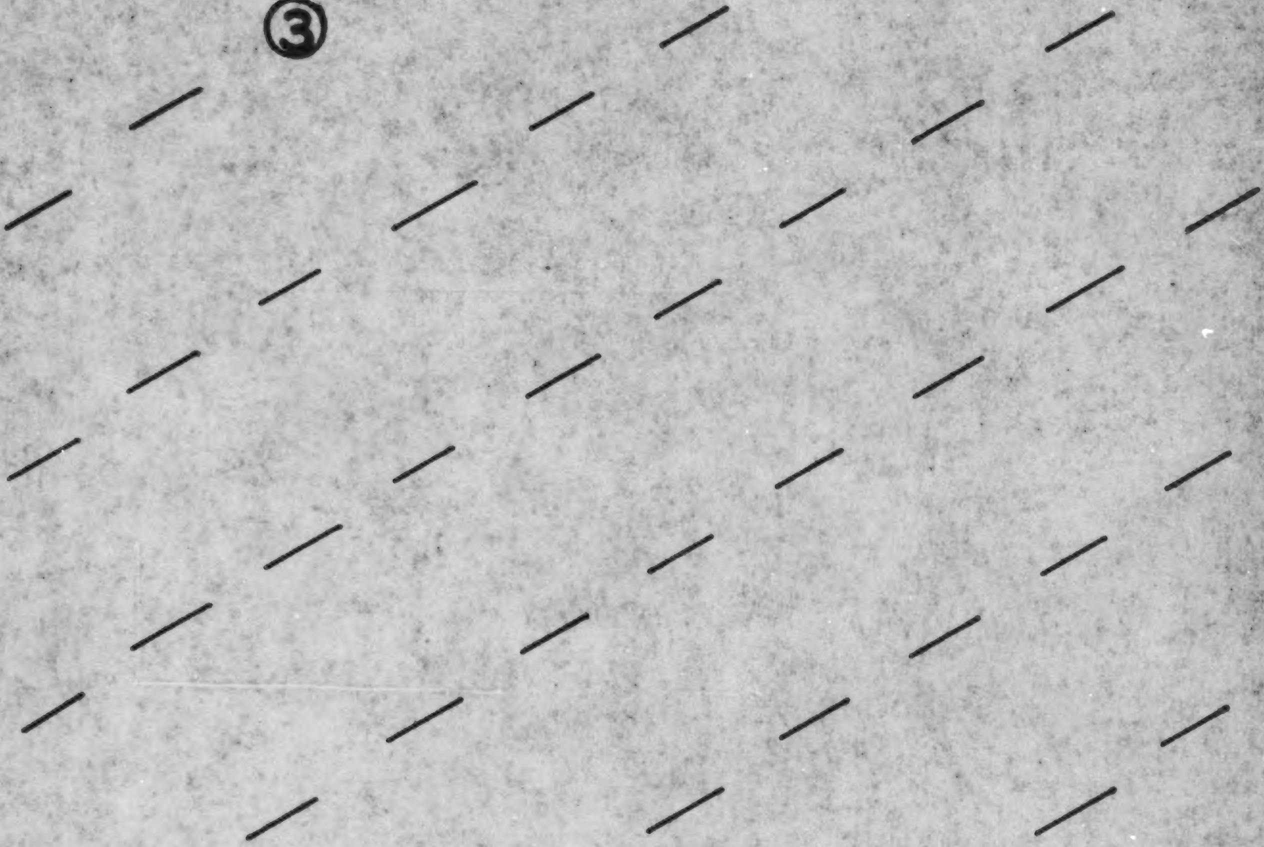
7 - 4 Left

②

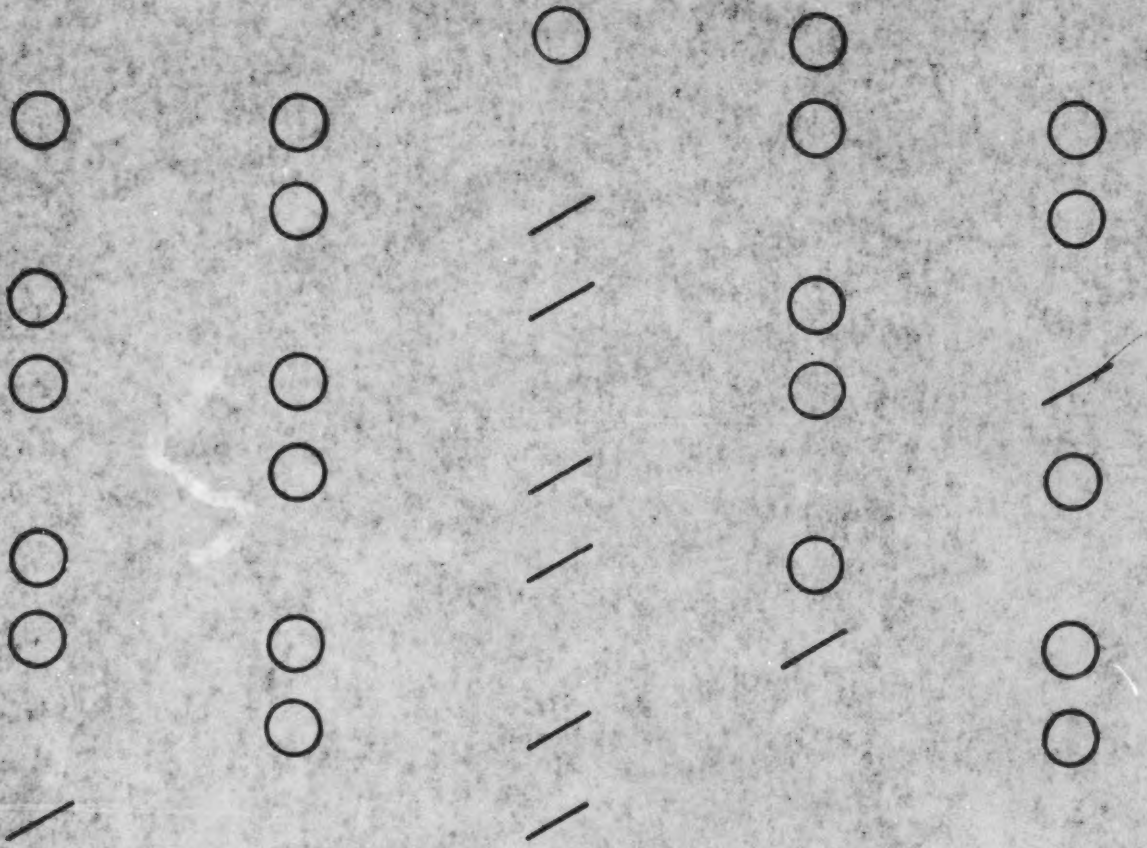


7 - 4 Right

③



7 - 4 Bottom



EXPANDED NOTATION

$$0.7245 = \left(7 \times \frac{1}{10}\right) + \left(2 \times \frac{1}{100}\right) + \left(4 \times \frac{1}{1000}\right) + \left(5 \times \frac{1}{10000}\right)$$

$$= \left(7 \times \frac{1}{10}\right) + \left(2 \times \frac{1}{10^2}\right) + \left(4 \times \frac{1}{10^3}\right) + \left(5 \times \frac{1}{10^4}\right)$$

$$= (7 \times 10^{-1}) + (2 \times 10^{-2}) + (4 \times 10^{-3}) + (5 \times 10^{-4})$$

$.3412 \quad (3 \times \frac{1}{5}) + (4 \times \frac{1}{25}) + (1 \times \frac{1}{125}) + (2 \times \frac{1}{625})$

FIVE

$(3 \times \frac{1}{5}) + (4 \times \frac{1}{5^2}) + (1 \times \frac{1}{5^3}) + (2 \times \frac{1}{5^4})$

$(3 \times 5^{-1}) + (4 \times 5^{-2}) + (1 \times 5^{-3}) + (2 \times 5^{-4})$

REPEATING DECIMALS (1)

$$
\begin{array}{r|c|c|c|c|c|c|c|c|c}
 & & . & 1 & 5 & 3 & 8 & 4 & 6 & 1 \\ \cline{2-10}
13\;) & 2 & . & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 1 & & 3 & & & & & & \\ \cline{2-4}
 & & & 7 & 0 & & & & & \\
 & & & 6 & 5 & & & & & \\ \cline{4-5}
 & & & & 5 & 0 & & & & \\
 & & & & 3 & 9 & & & & \\ \cline{5-6}
 & & & & 1 & 1 & 0 & & & \\
 & & & & 1 & 0 & 4 & & & \\ \cline{5-7}
 & & & & & & 6 & 0 & & \\
 & & & & & & 5 & 2 & & \\ \cline{7-8}
 & & & & & & & 8 & 0 & \\
 & & & & & & & 7 & 8 & \\ \cline{8-9}
 & & & & & & & & 2 & 0 \\
 & & & & & & & & 1 & 3 \\ \cline{9-10}
\end{array}
$$

$\frac{2}{13}$ .153846153846

$$\begin{array}{r}
70 \\
\underline{65} \\
50 \\
\underline{39} \\
110 \\
\underline{104} \\
6
\end{array}$$

REPEATING DECIMALS (2)

$$N = .213\overline{213}\ .\ .\ .$$

$$\begin{array}{rl} 1000N = & 213.213\overline{213}\ \ldots \\ \hline 999N = & 213.00000\overline{0}\ \ldots \end{array}$$

$$N = \frac{213}{999} = \frac{71}{333}$$

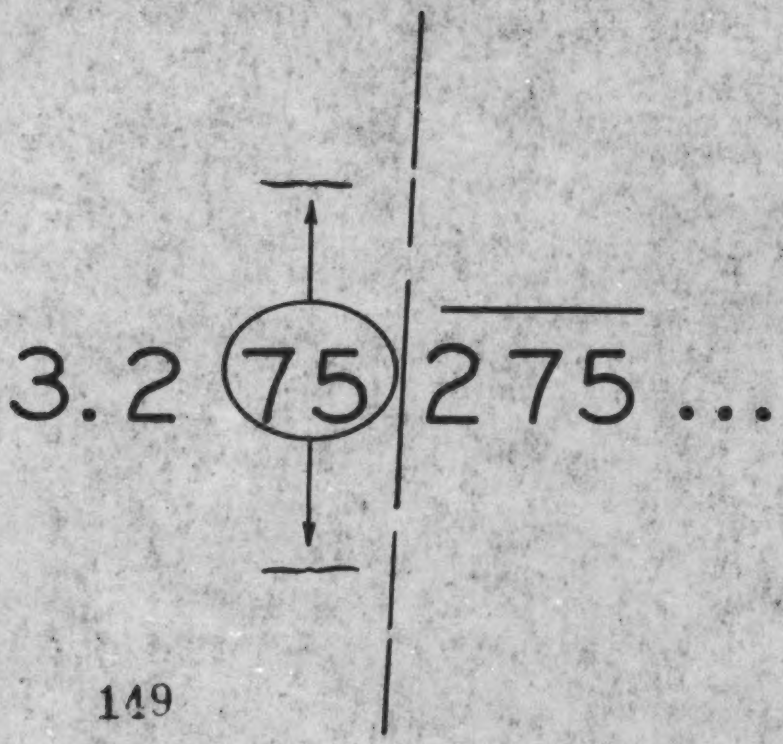
9 - 2 Right

$$\underline{N} = \underline{.47\overline{47}\ .\ .\ .}$$

LOCATING A RATIONAL AND AN IRRATIONAL NUMBER BETWEEN TWO RATIONAL NUMBERS.

$$3.2727\overline{27}\ldots$$

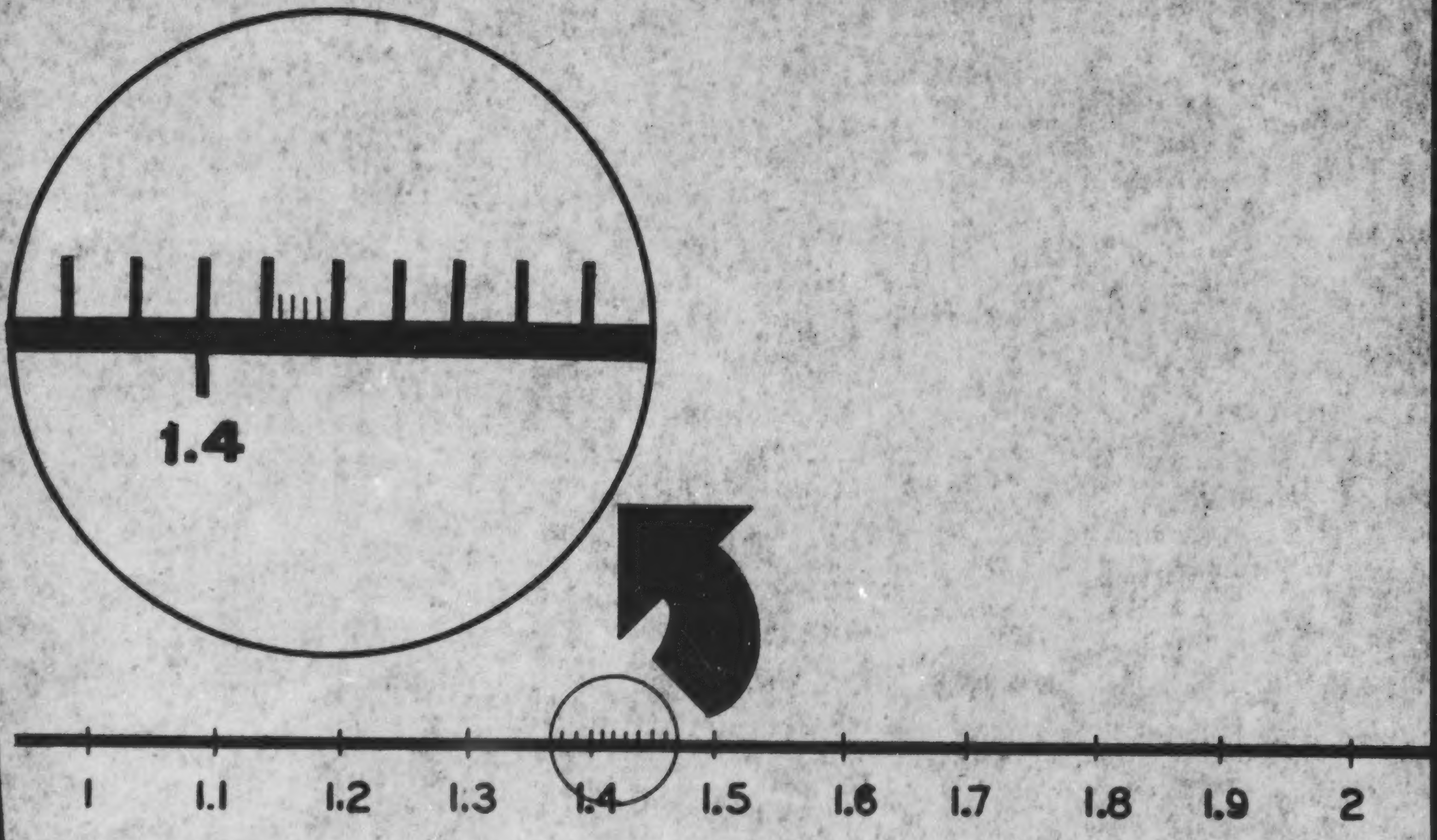
$$3.2828\overline{28}\ldots$$



RATIONAL : $3.2\,75\,\overline{275}\ldots$

IRRATIONAL : 3.2 75 275527555...

LOCATING $\sqrt{2}$

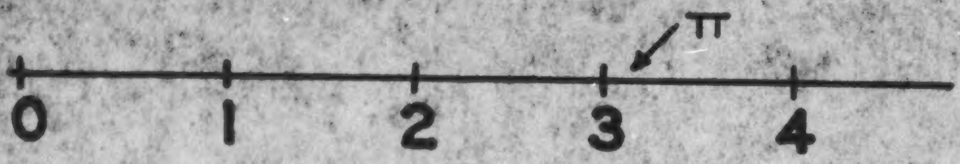

1.4
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

BETWEEN 1 AND 2
WE CAN LOCATE 1.4

BETWEEN 1.4 AND 1.5
WE CAN LOCATE 1.41

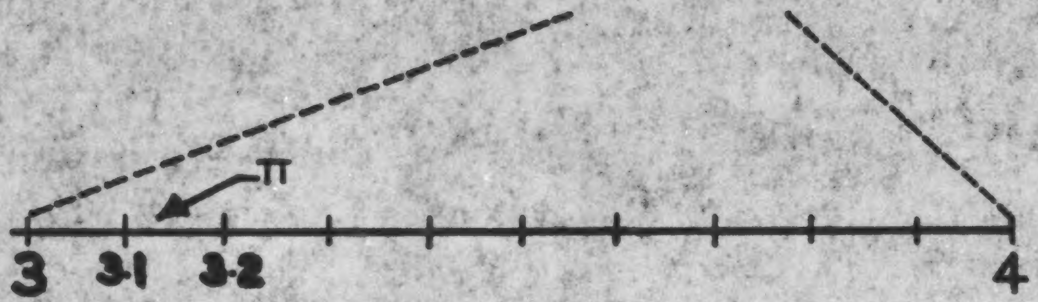
9 - 4 Bottom

BETWEEN 1.41 & 1.42
WE CAN LOCATE 1.414

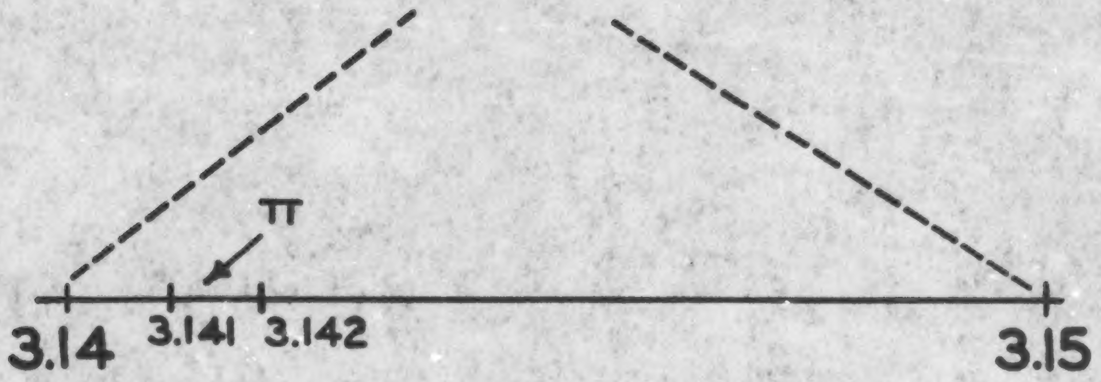


$3 < \pi < 4$

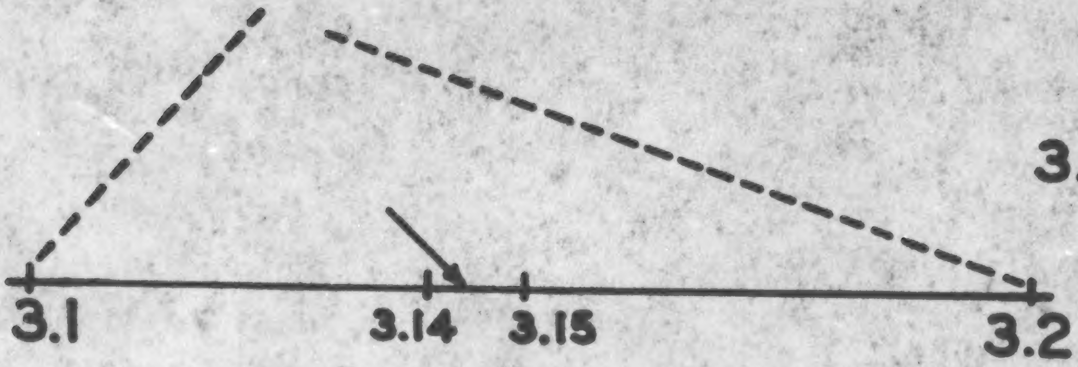
LOCATING π ON THE NUMBER LINE



$3.1 < \pi < 3.2$


π
3.14
3.141
3.142
3.15

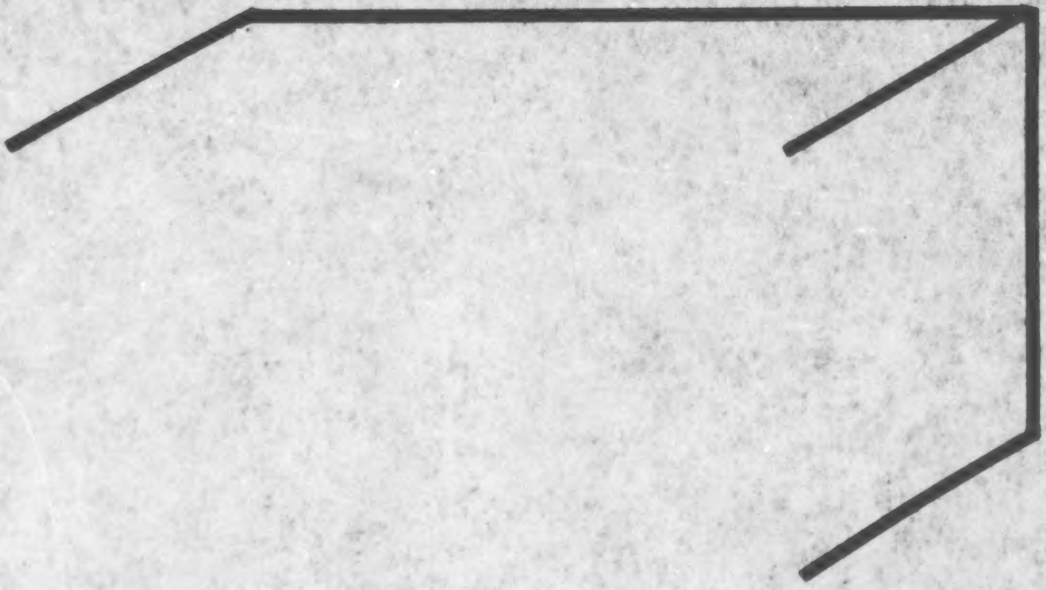
$3.141 < \pi < 3.142$


3.1
3.14
3.15
3.2

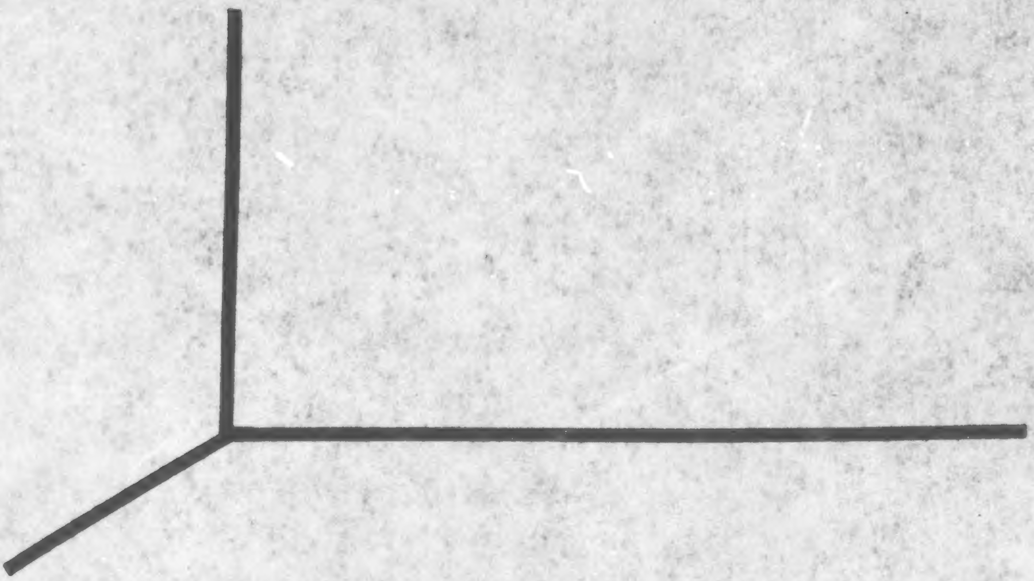
$3.14 < \pi < 3.15$

A
D
B
C

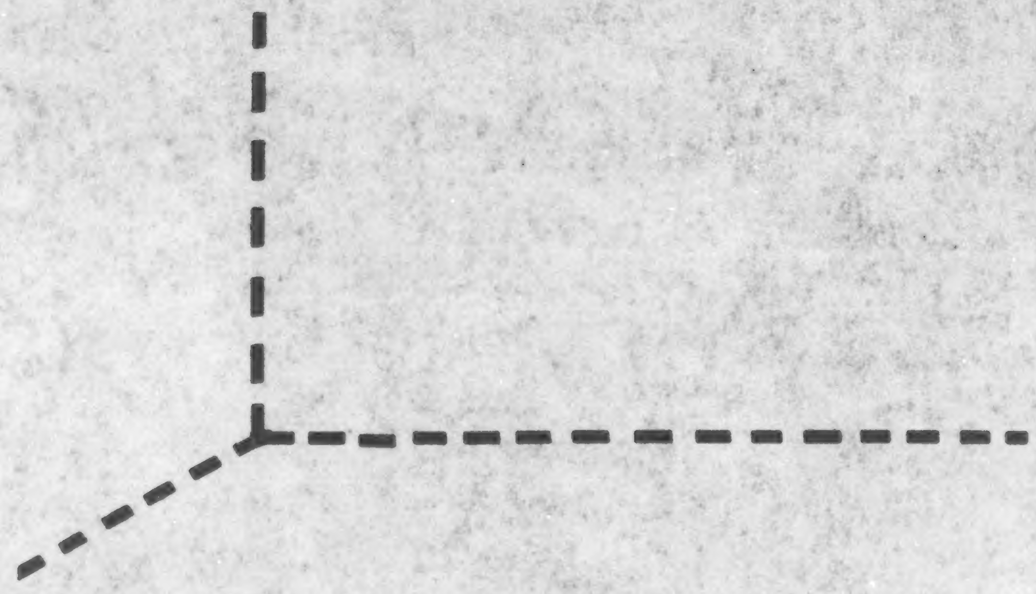
10 - 1 Left

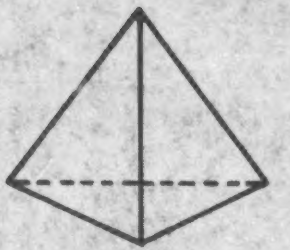


10 - 1 Right

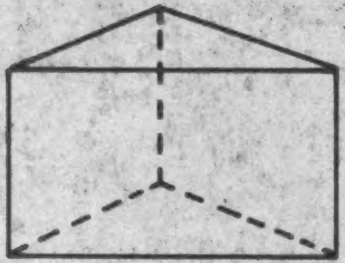


10 - 1 Bottom

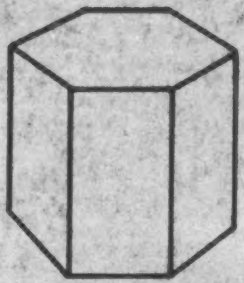




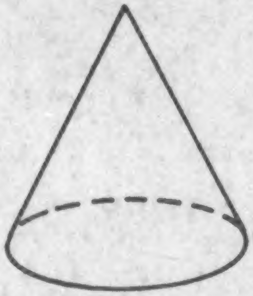
TETRAHEDRON



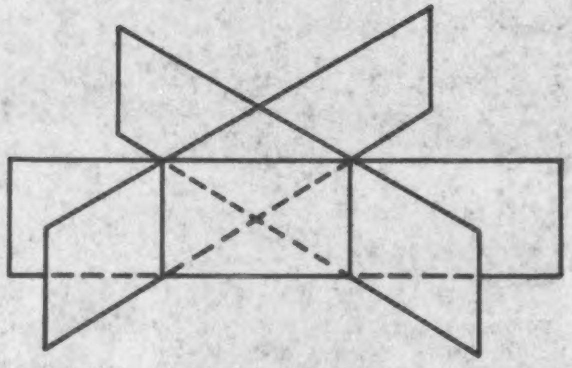
RIGHT TRIANGULAR PRISM



RIGHT HEXAGONAL PRISM



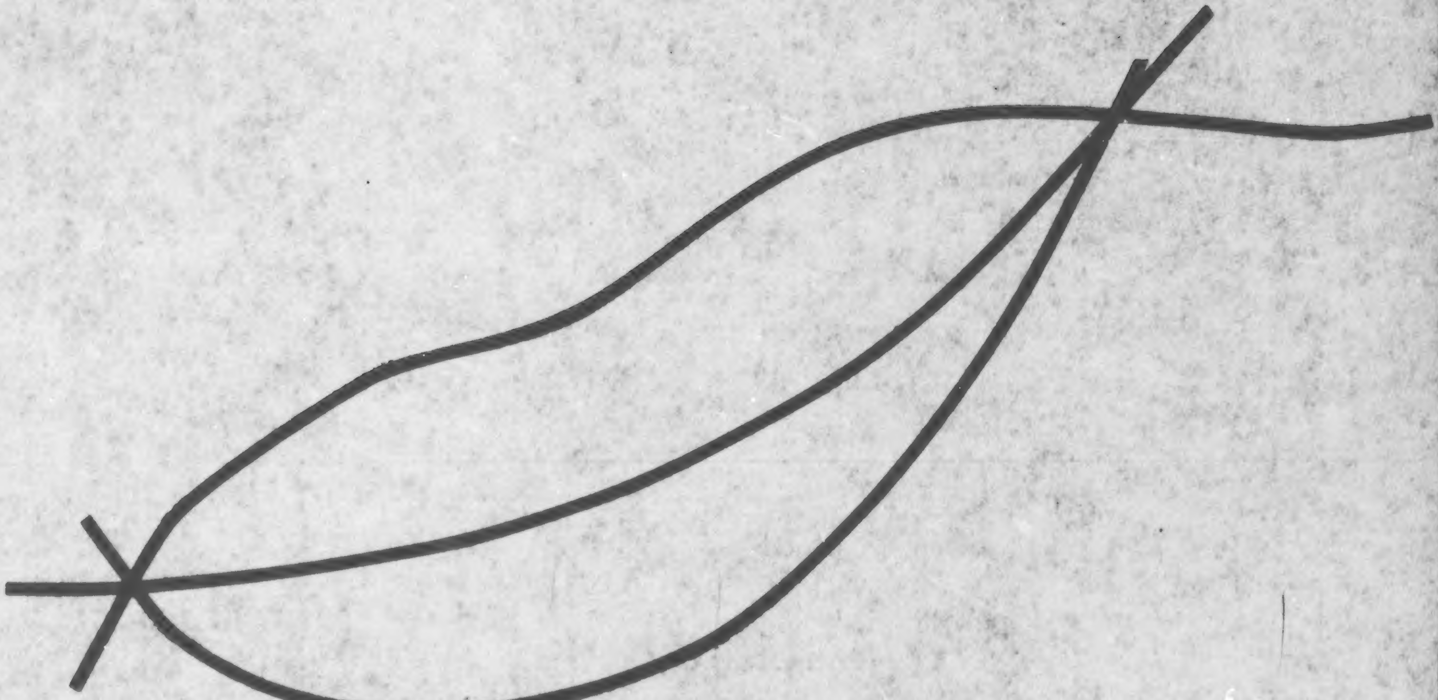
RIGHT CIRCULAR CONE



THREE INTERSECTING PLANES

PROPERTY I: THROUGH ANY TWO POINTS IN SPACE THERE IS EXACTLY ONE LINE.

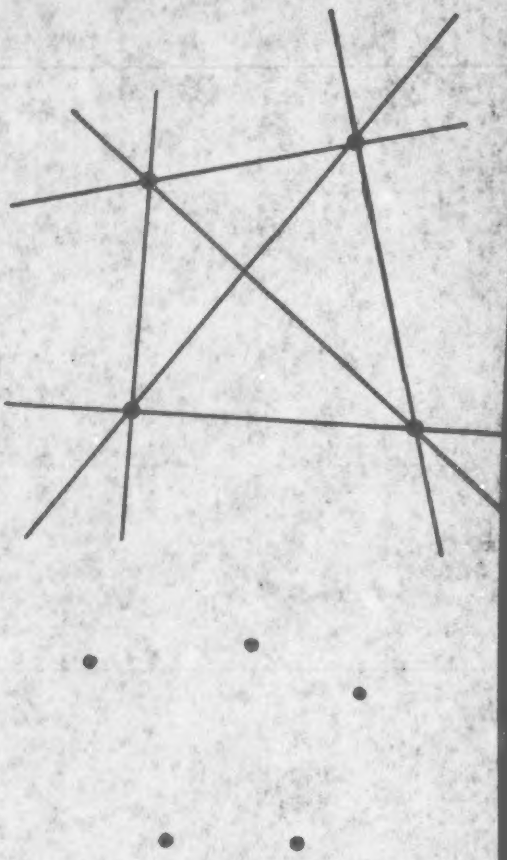
10 - 3 Left



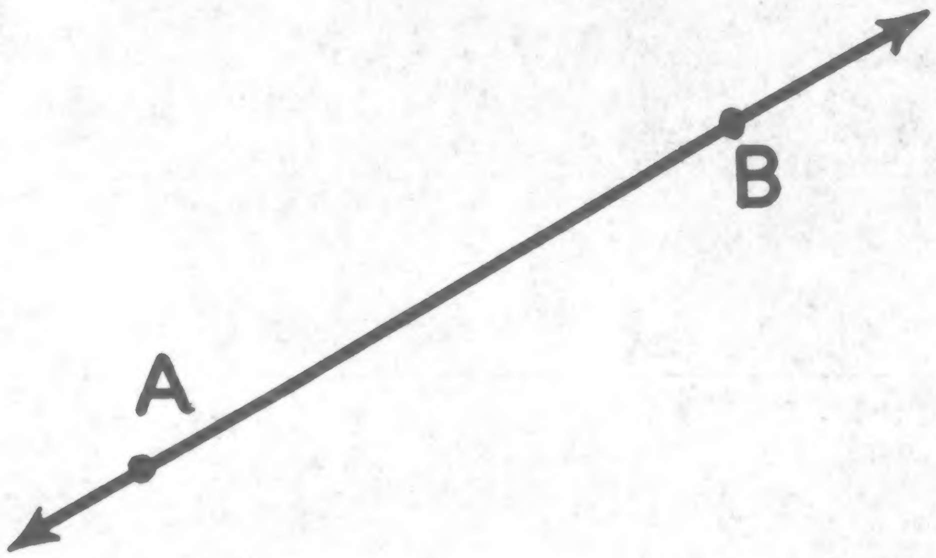
10 - 3 Right

LINES DETERMINED BY POINTS

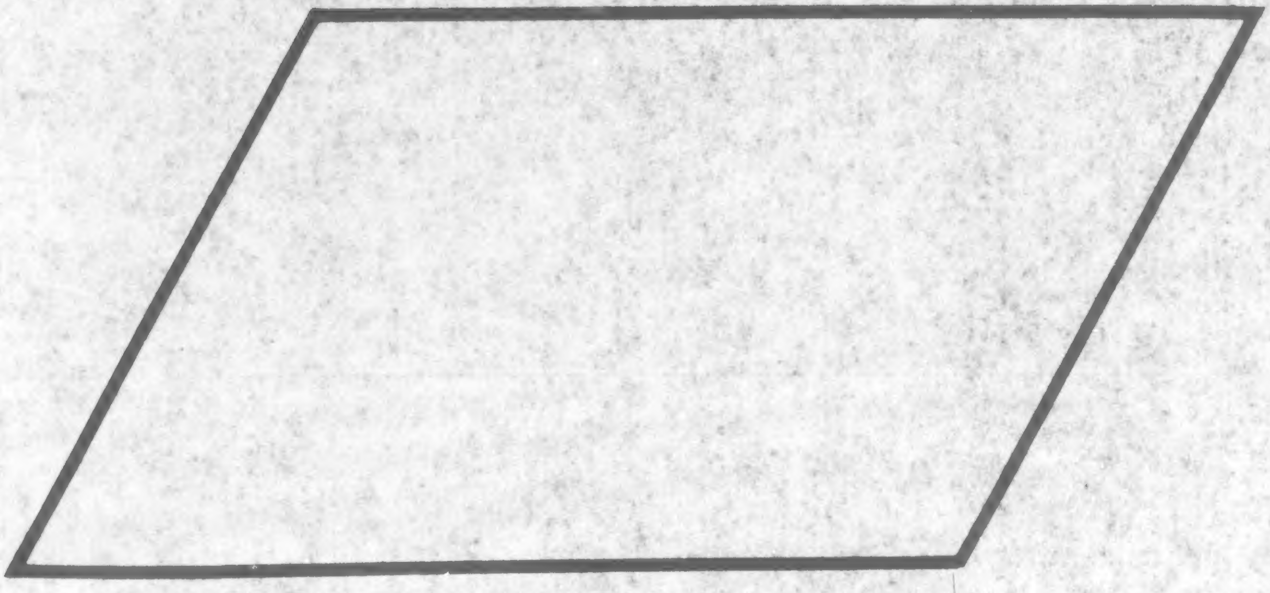
NUMBER OF POINTS	NUMBER OF LINES
2	
3	
4	
5	
6	
n	



PROPERTY 2: IF A LINE CONTAINS TWO DIFFERENT POINTS OF A PLANE,

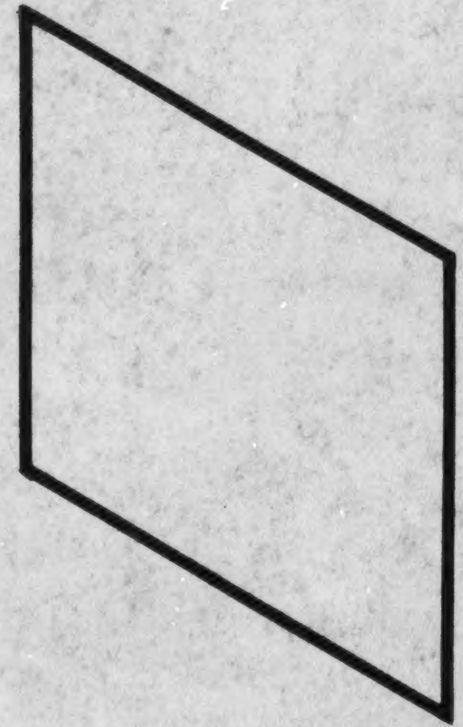


THEN THE LINE LIES IN THE PLANE.



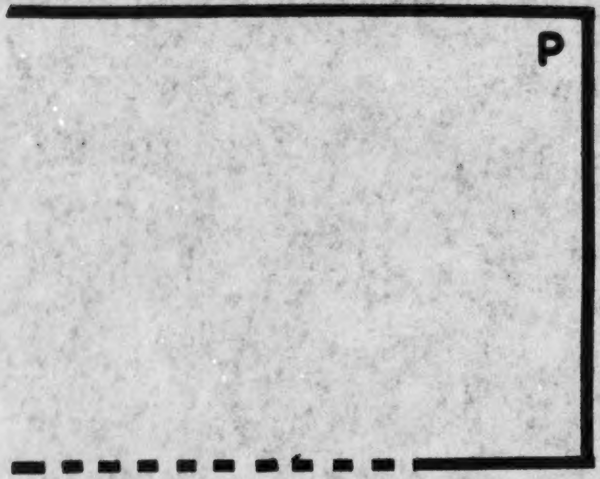
PROPERTY 3: ANY THREE POINTS NOT IN THE SAME STRAIGHT LINE ARE IN EXACTLY ONE PLANE.

10 - 6 Left

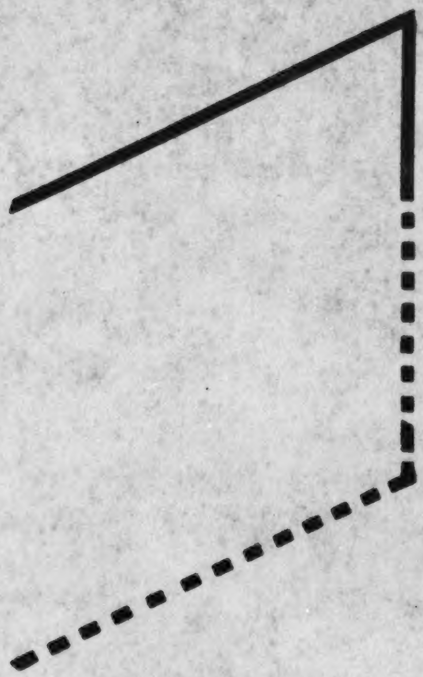


179

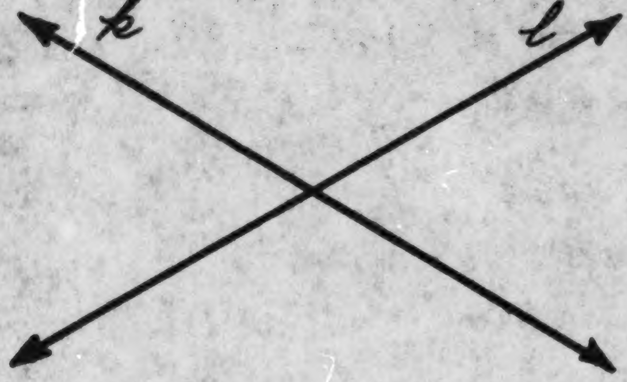
10 - 6 Right



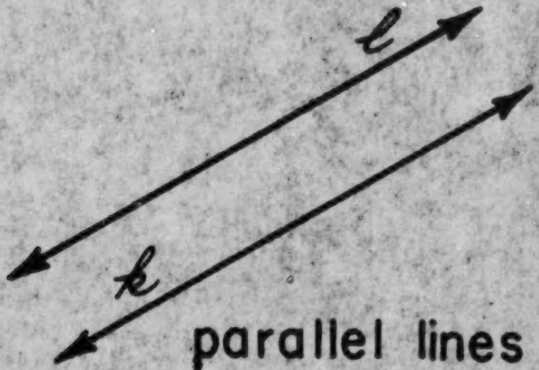
10 - 6 Bottom



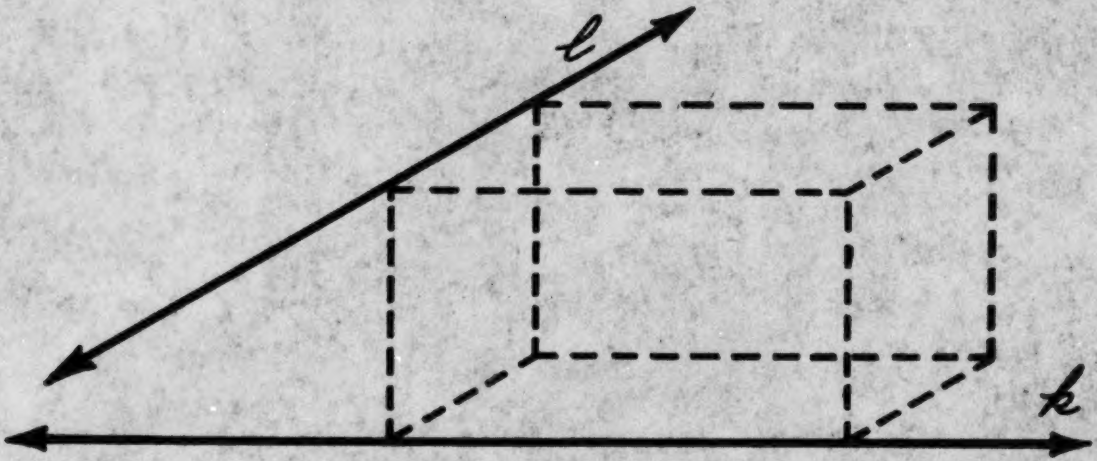
TWO LINES IN SPACE



intersecting lines

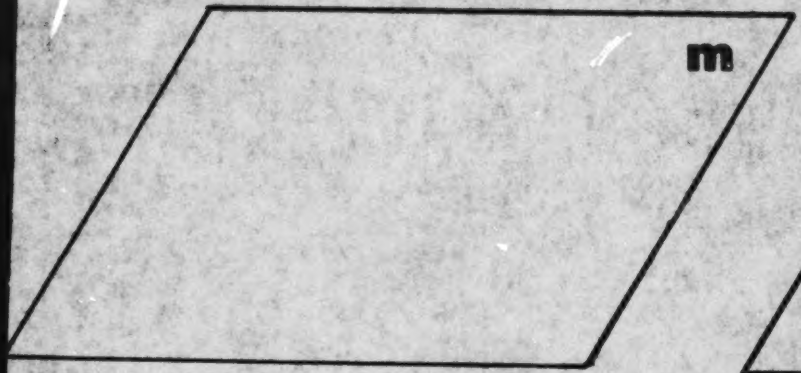


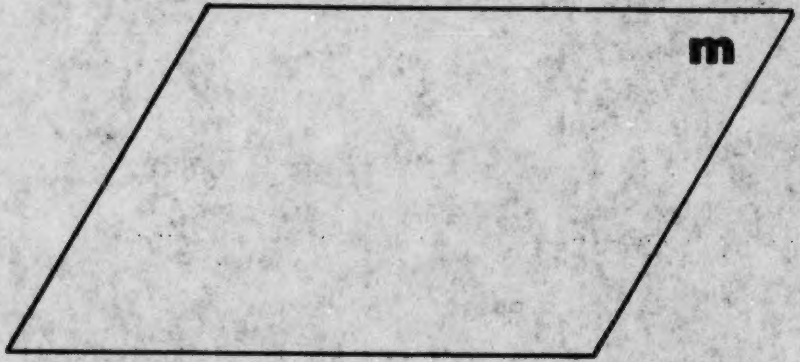
parallel lines

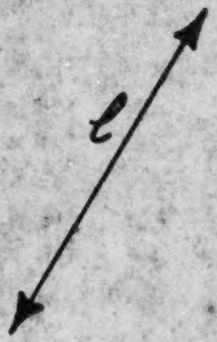


SKEW LINES

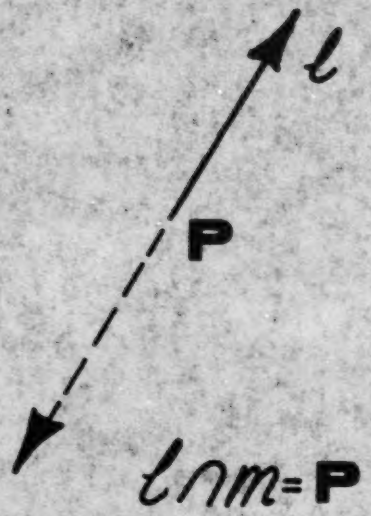
A LINE AND A PLANE IN SPACE

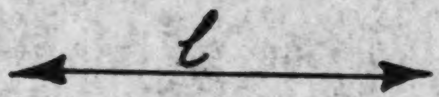






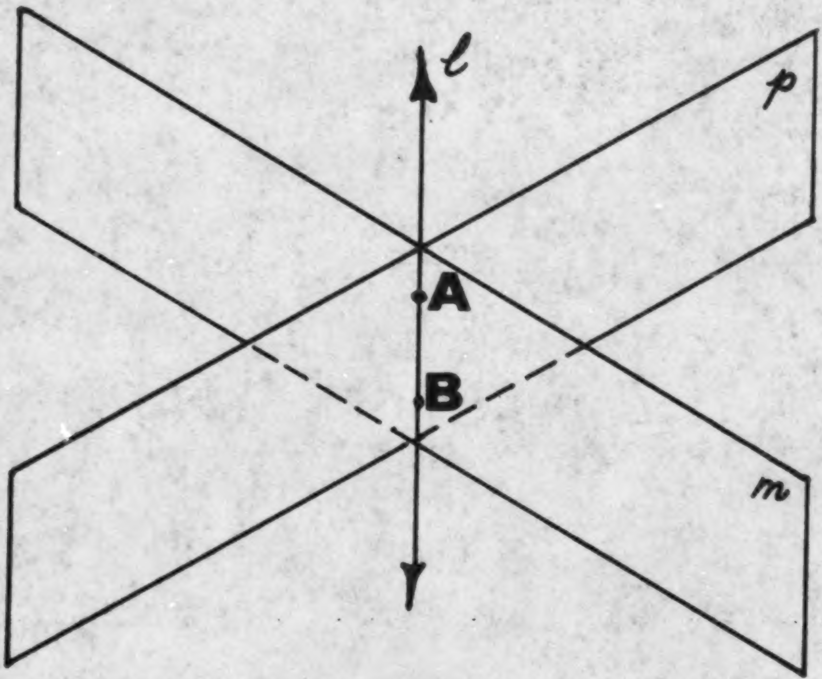
$\ell \cap m = \ell$


l
P
$l \cap m = P$

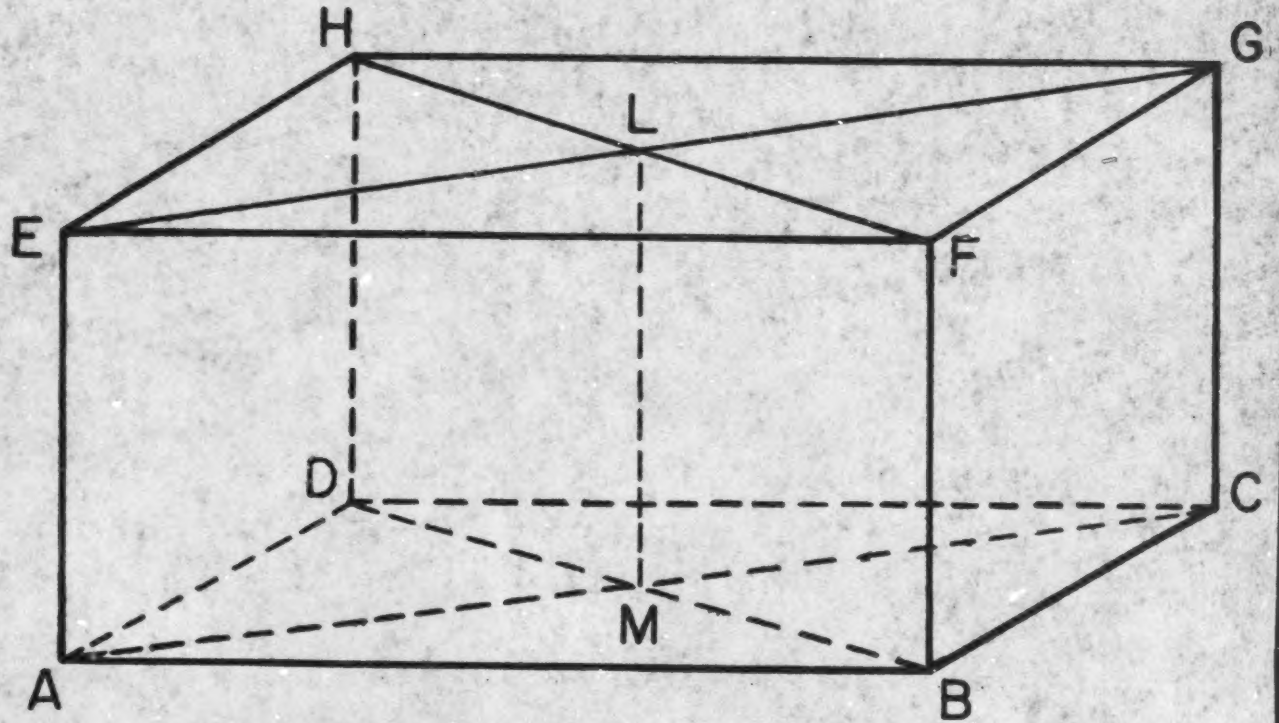


$$\ell \cap m = \emptyset$$

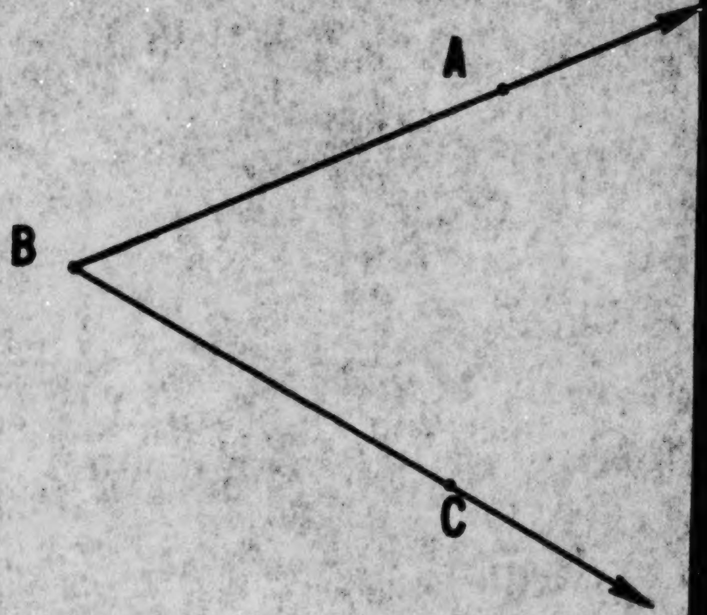
PROPERTY 4: IF THE INTERSECTION OF TWO DIFFERENT PLANES IS NOT EMPTY THEN THE INTERSECTION IS A LINE.



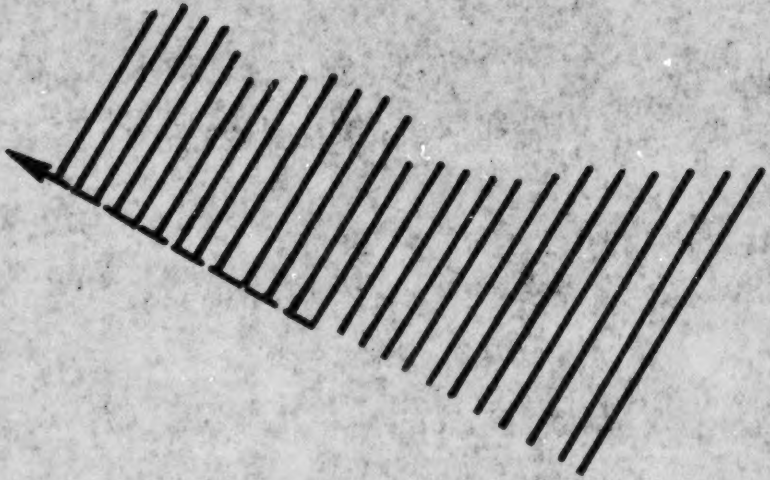
PLANES IN SPACE


H
G
L
E
F
D
C
M
A
B

INTERIOR OF ANGLE ABC ($\angle ABC$)

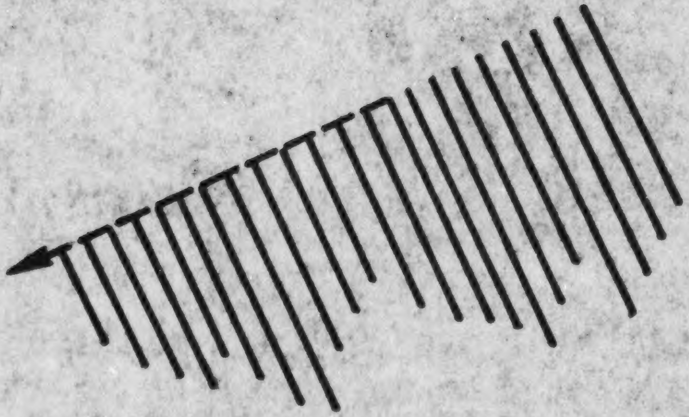


HOW WOULD YOU DEFINE "INTERIOR OF $\angle ABC$" ?



A-SIDE OF $\overleftrightarrow{BC}$

11 - 1 Right

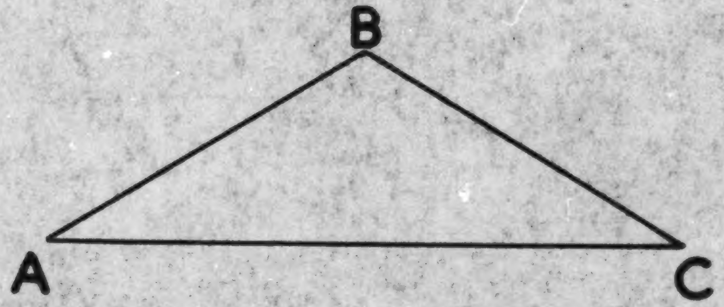


$\cap$ C-SIDE OF $\overleftrightarrow{AB}$

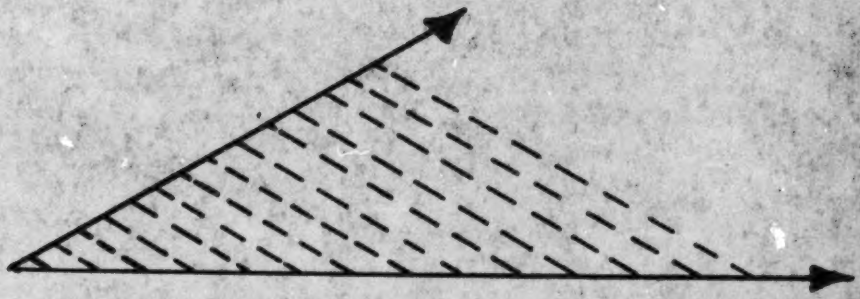
11 - 1 Bottom

DOES THIS AGREE WITH YOUR DEFINITION?

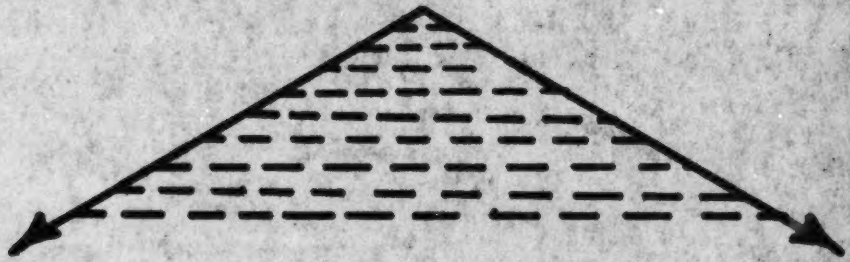
INTERIOR OF TRIANGLE ABC ($\triangle ABC$)



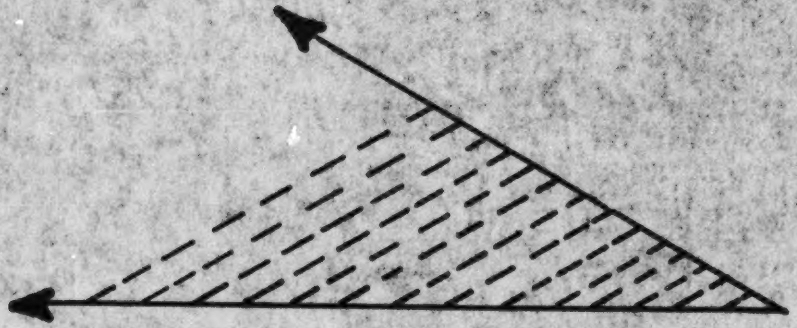
HOW WOULD YOU DEFINE "INTERIOR OF $\triangle ABC$?"



INTERIOR ∠A



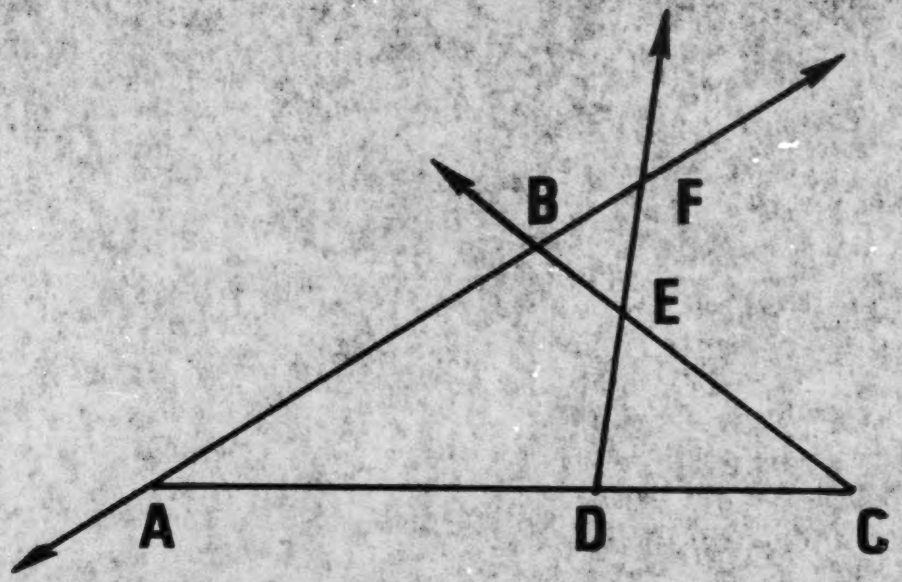
$\cap$ INTERIOR $\angle B$



$\cap$ INTERIOR $\angle C$

COULD WE USE THE INTERSECTION OF ANY TWO OF THESE SETS TO DEFINE THE INTERIOR ?

LINES, ANGLES, RAYS, AND SEGMENTS



$\overleftrightarrow{AB} \cap \triangle ABC = \overline{AB}$

$\overrightarrow{BA} \cap \overline{BC} = B$

$\overrightarrow{BA} \cup \overline{BF} = $ _____

$\overrightarrow{BA} \cup \overrightarrow{BF} = $ _____

$\overrightarrow{EB} \cup \overrightarrow{EF} = $ _____

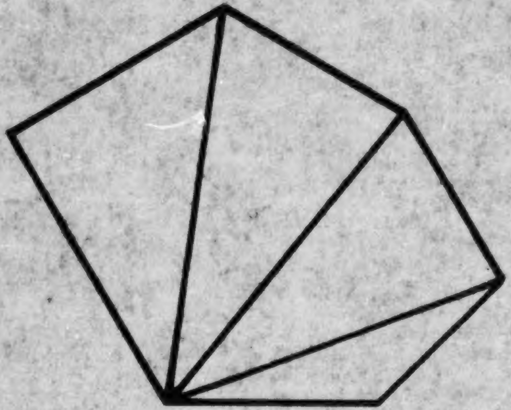
$\triangle ABC \cap \overrightarrow{DE} = $ _____

$\triangle EBF \cap \triangle DEC = $ _____

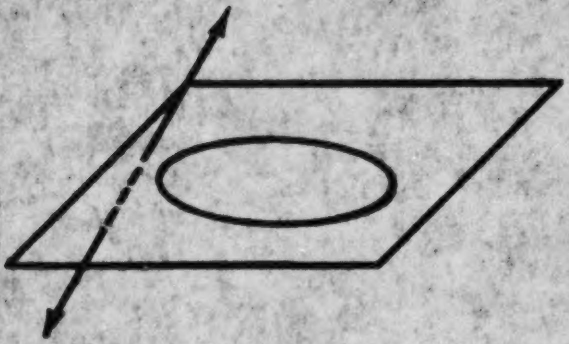
$\triangle DEC \cap \overleftrightarrow{AB} = $ _____

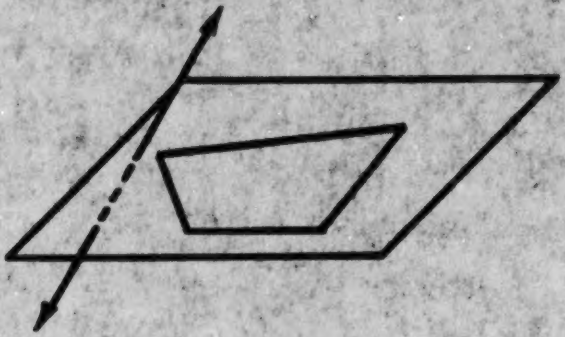
DIAGONALS OF POLYGONS

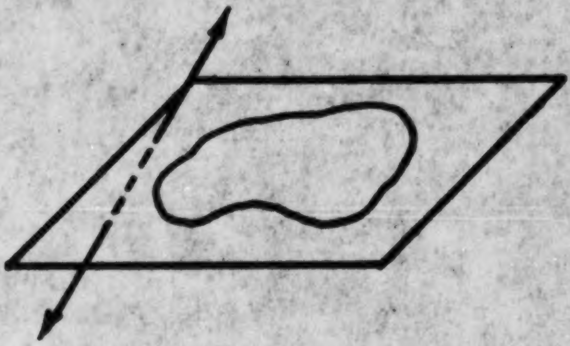
NUMBER OF SIDES	NUMBER OF DIAGONALS
3	0
4	2
5	5
6	
7	
8	
⋮	
n	



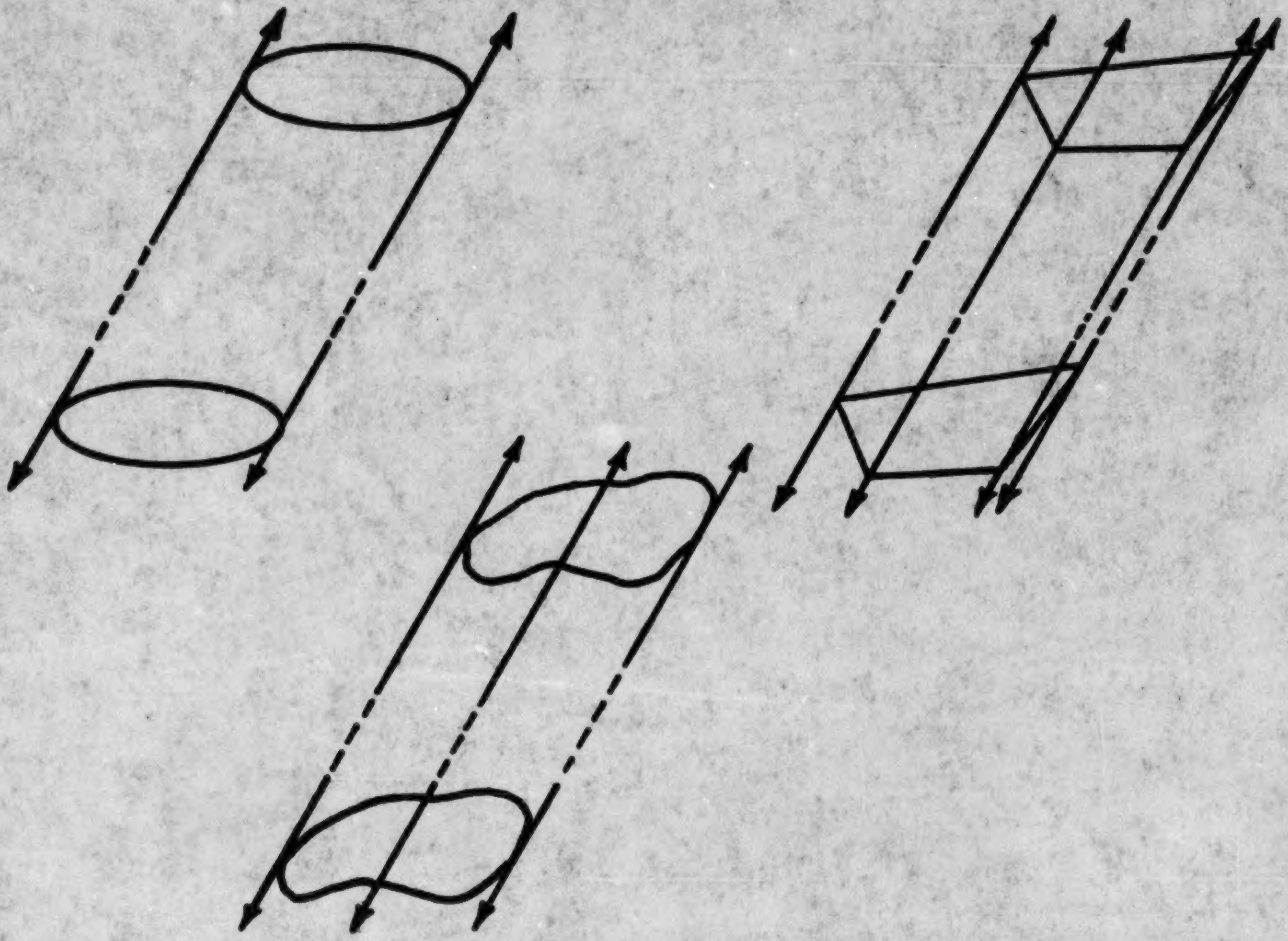
CYLINDRICAL SURFACES

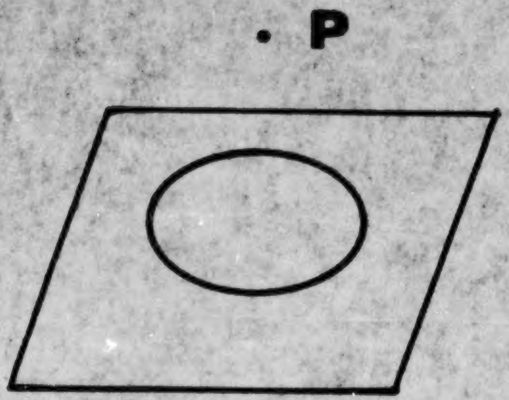


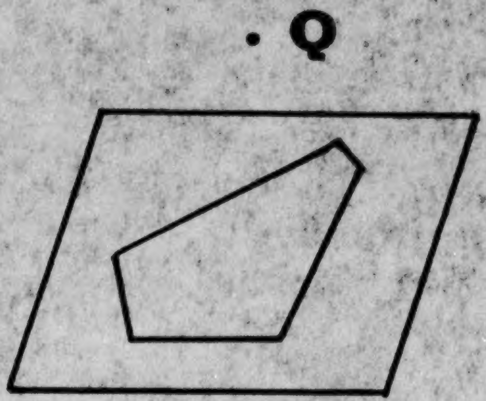


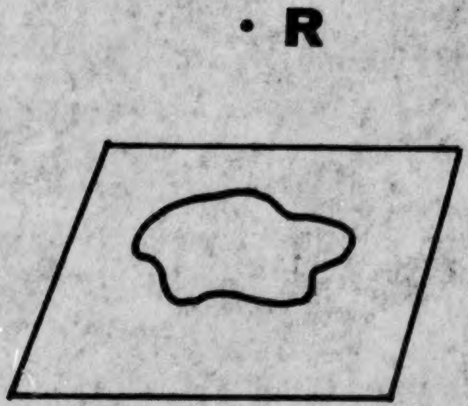


11 - 5 Left



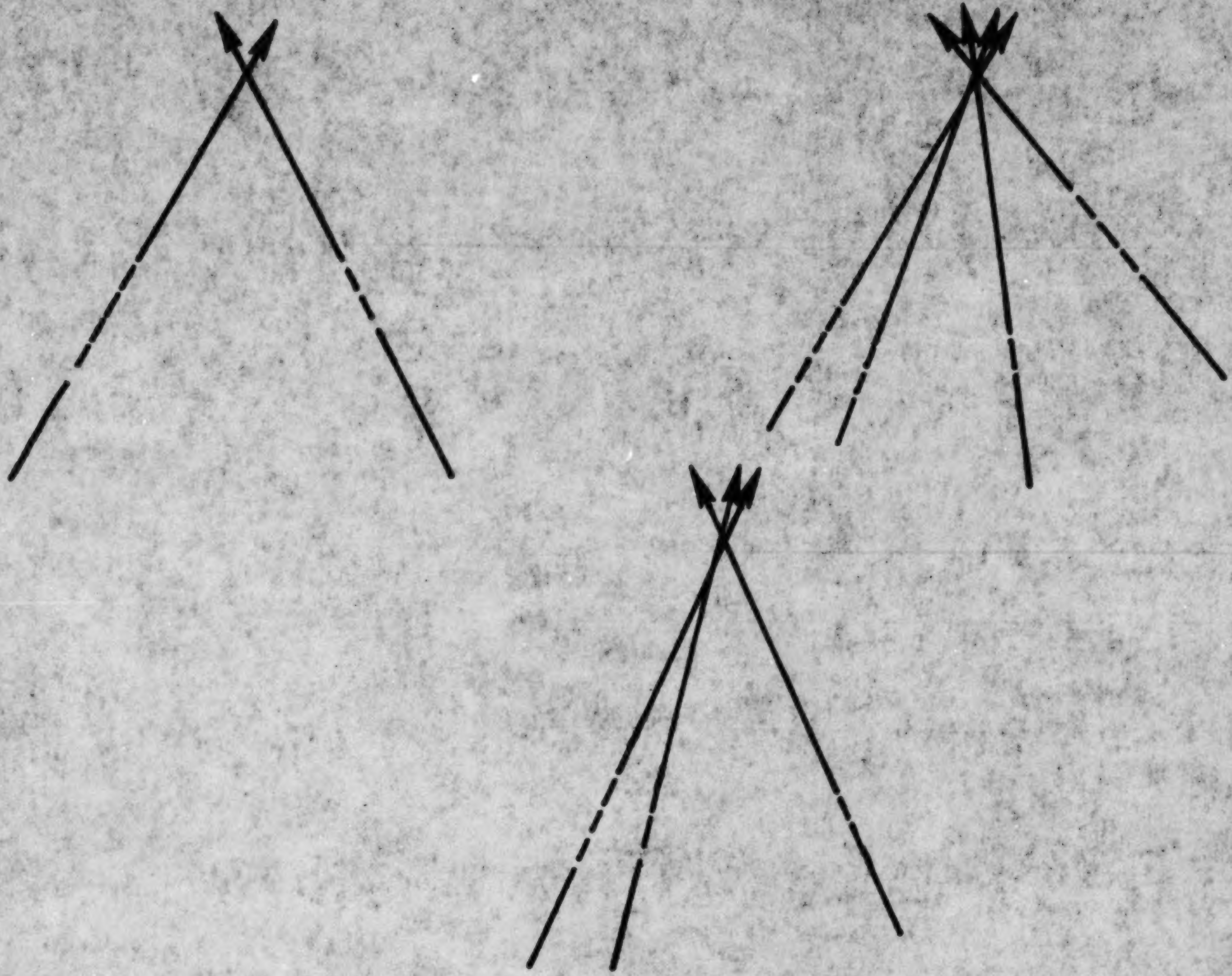




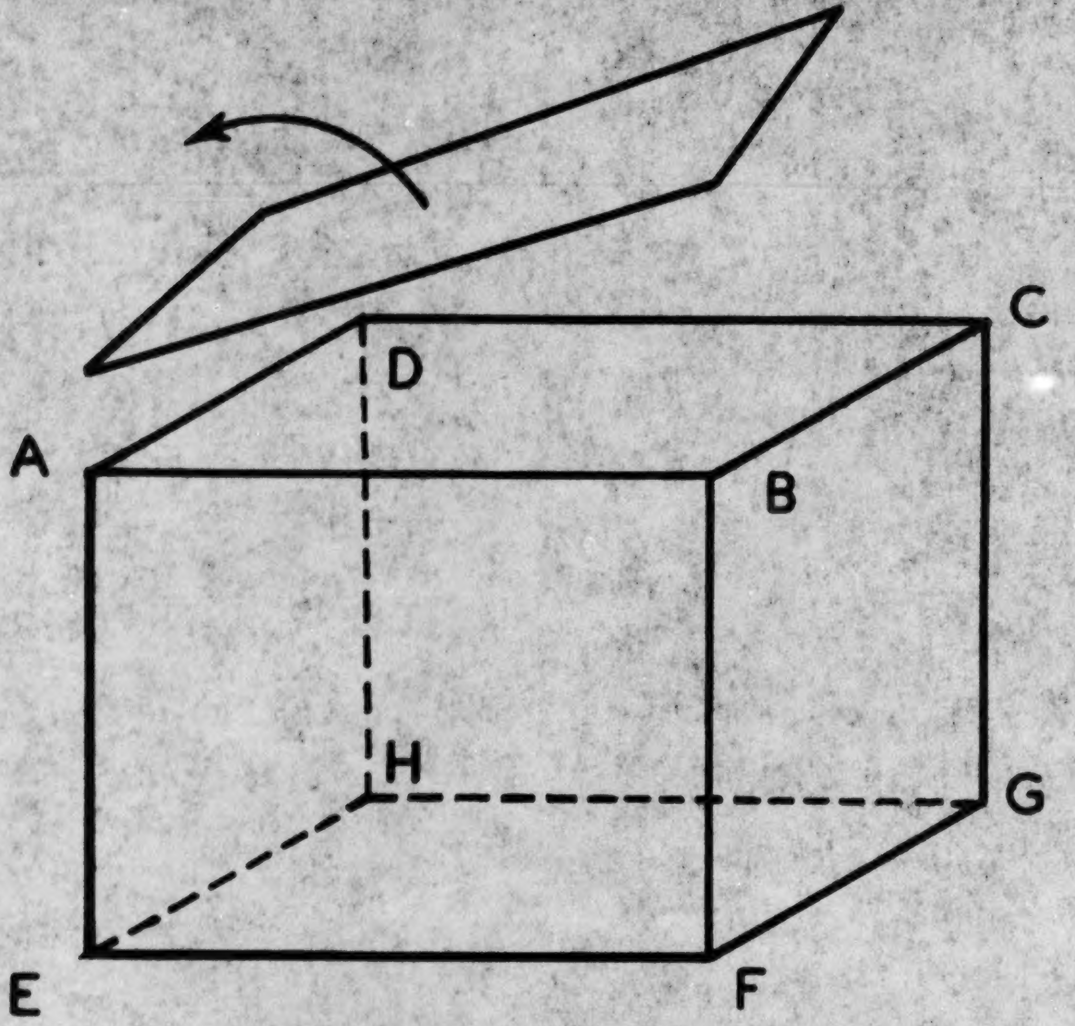


CONICAL SURFACES

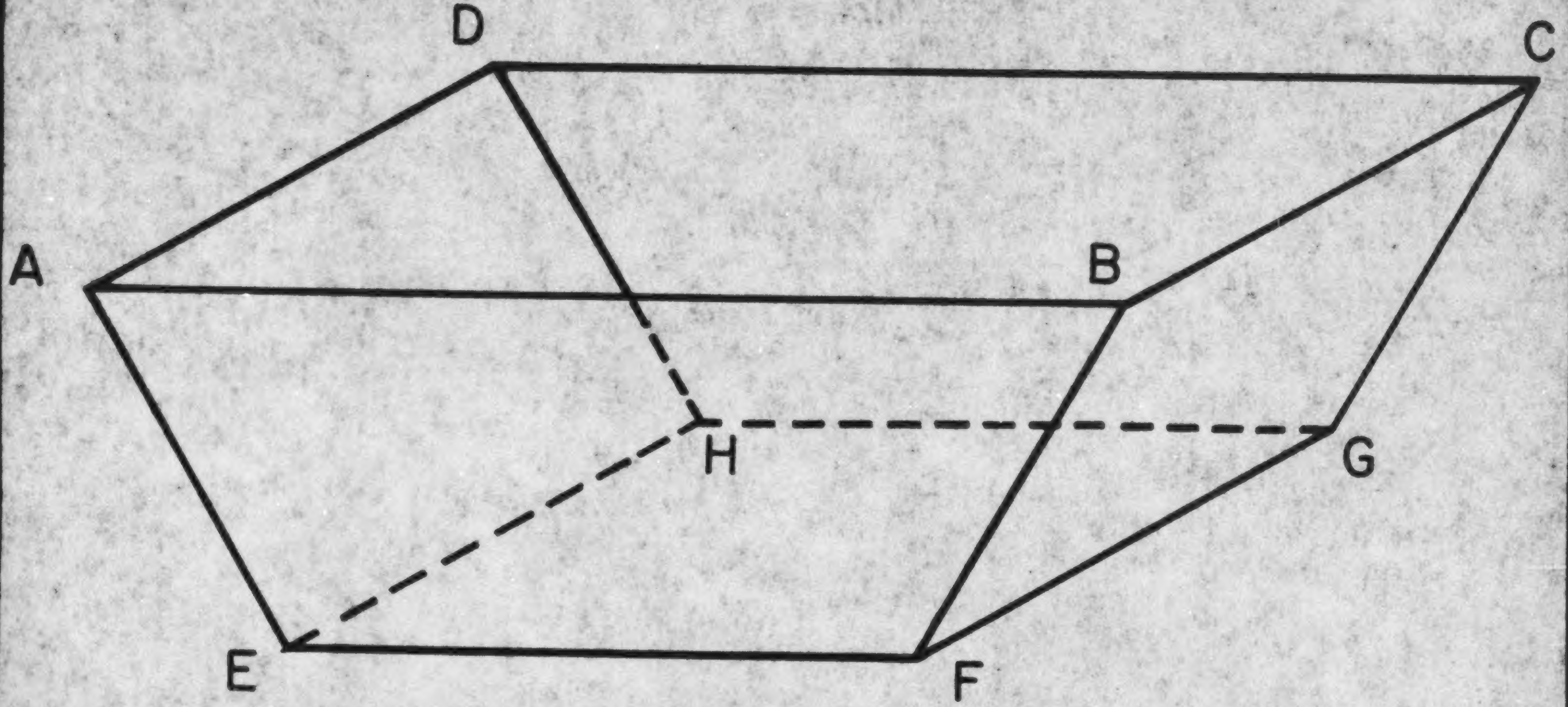
11 - 6 Left

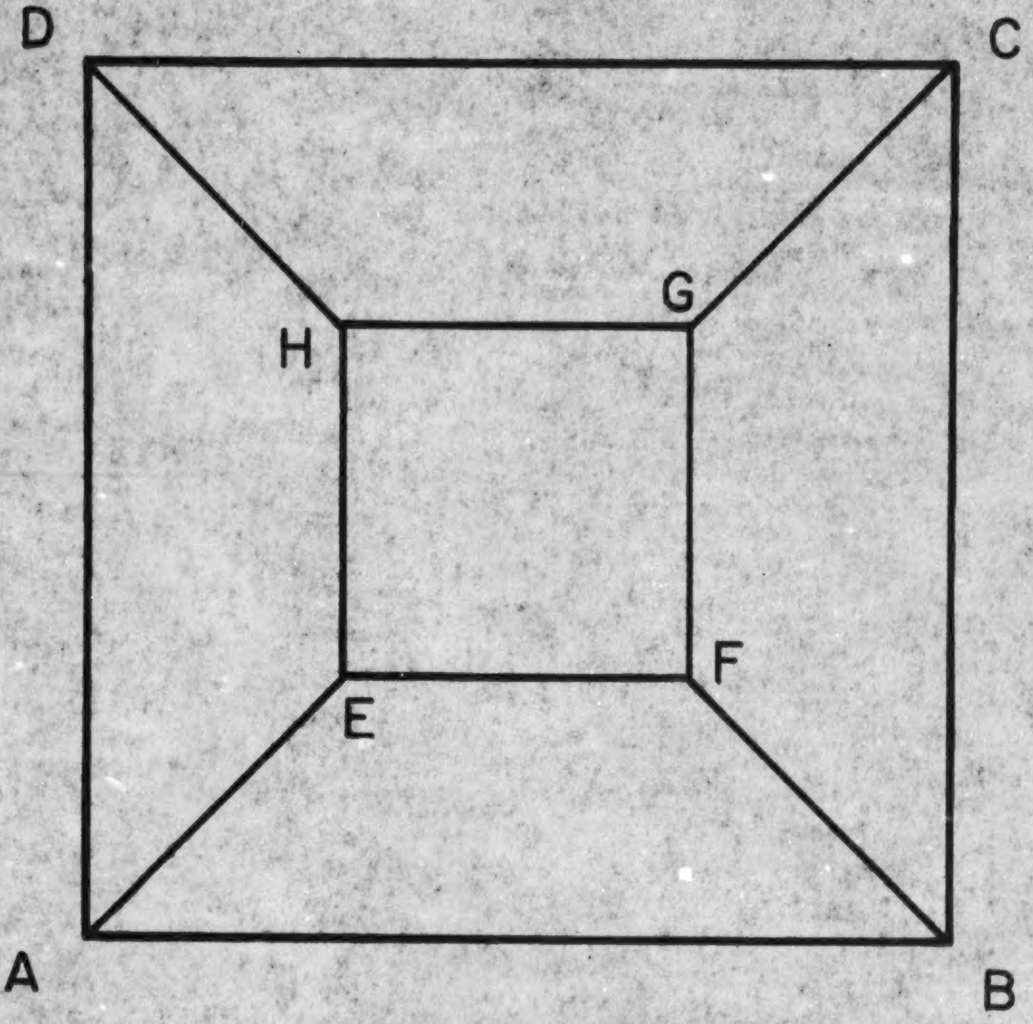


$$V - E + F = 2 \qquad (1)$$

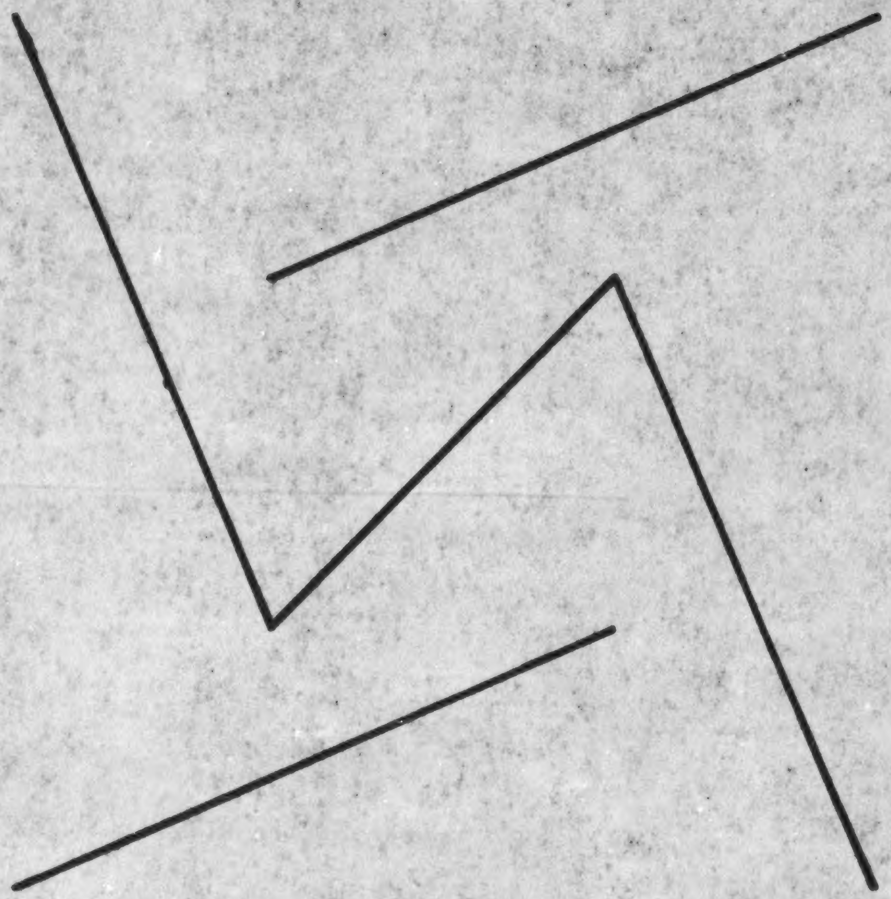


$V - E + F = 2 \quad (2)$

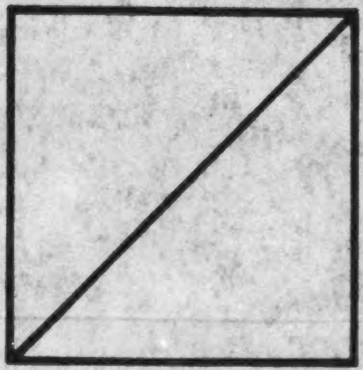



D
C
G
H
F
E
A
B

$V - E + F = 2 \quad (3)$



$V - E + F = 2$ (4)

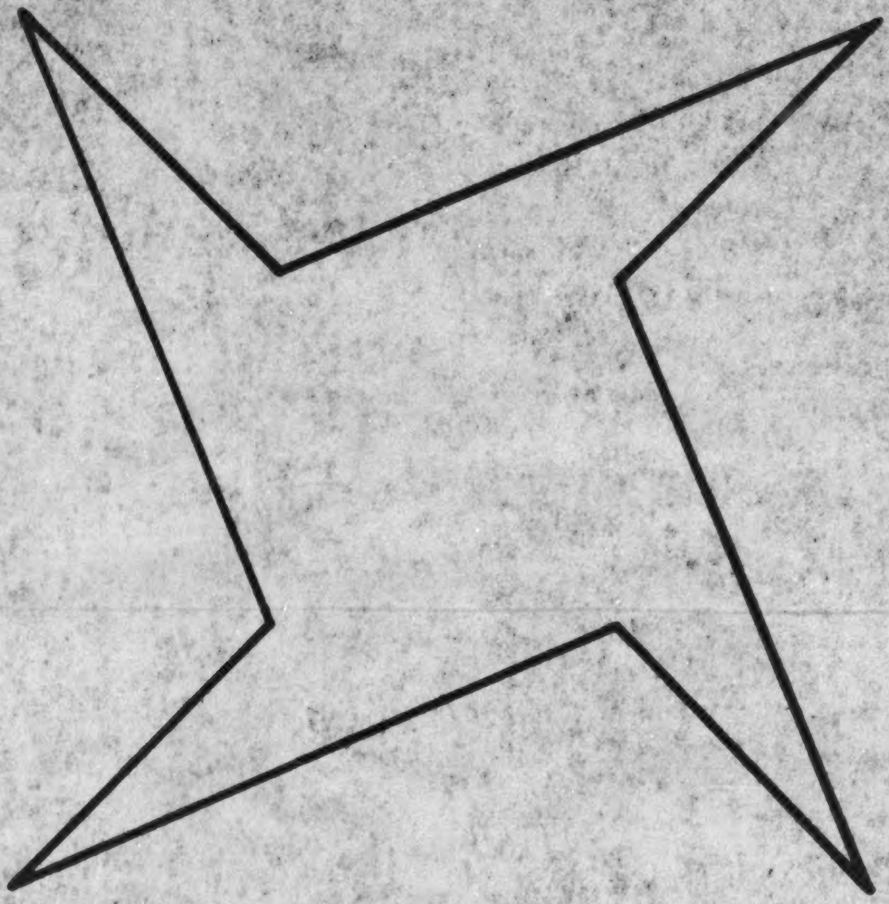


The overlays for transparency 11 - are to be removed in the order, LEFT, RIGHT, BOTTOM.

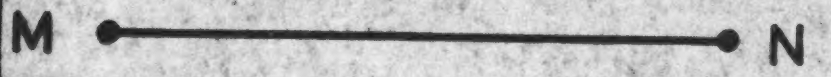
11 - 10 Left

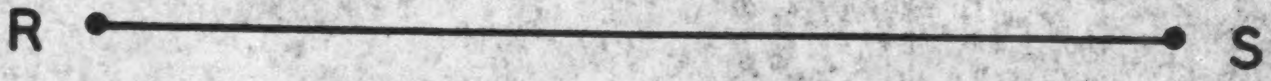
11 - 10 Right

11 - 10 Bottom

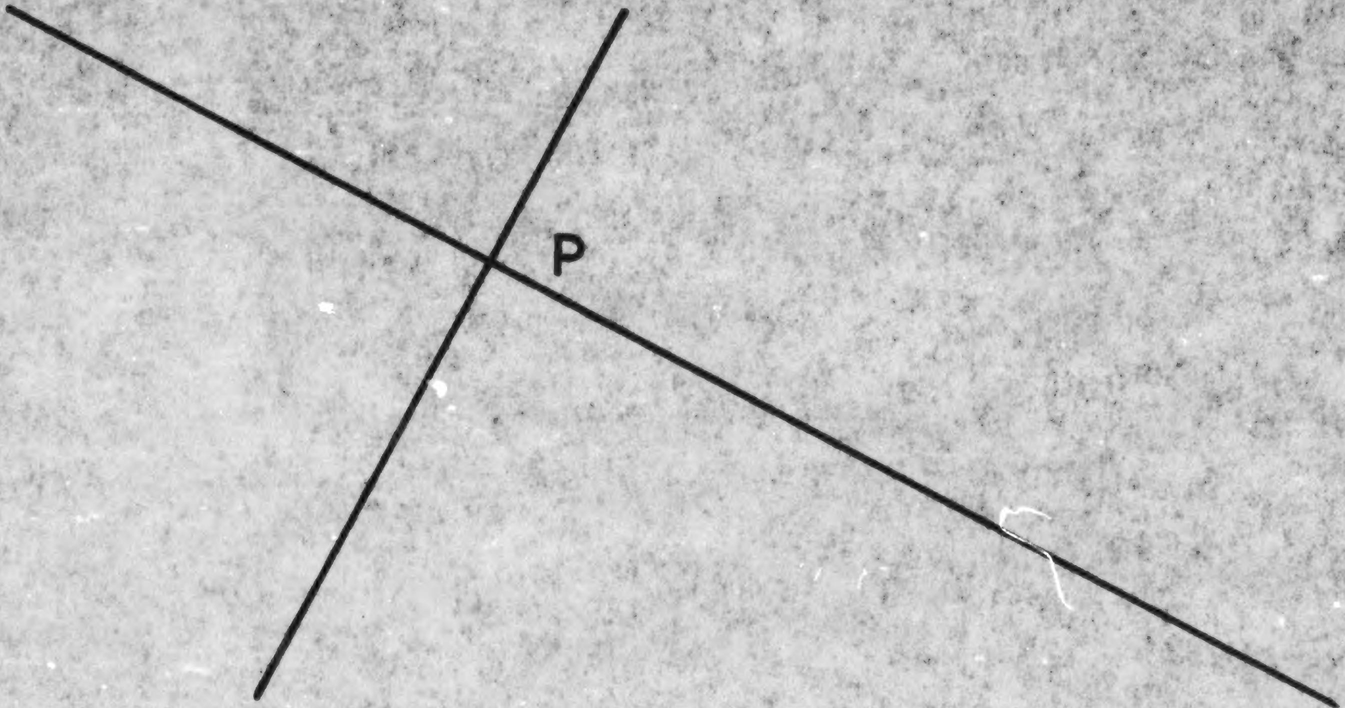


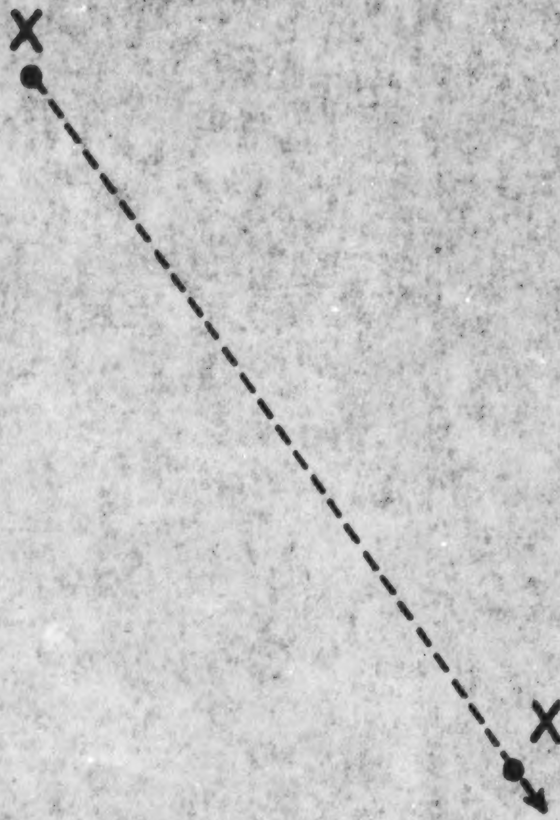
ONE-TO-ONE CORRESPONDENCE



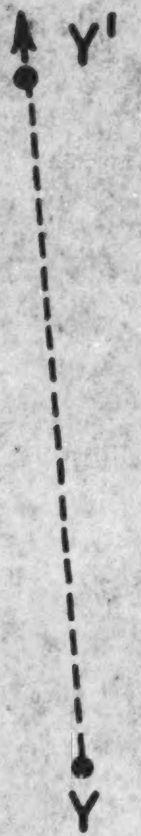


12 - 1 Left



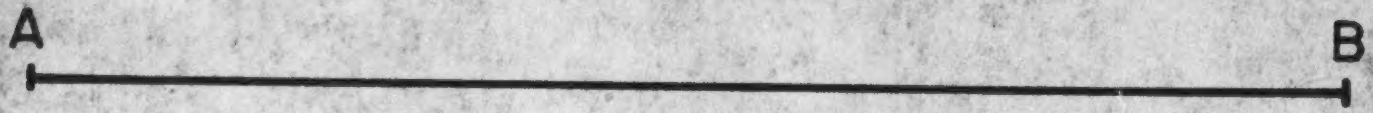


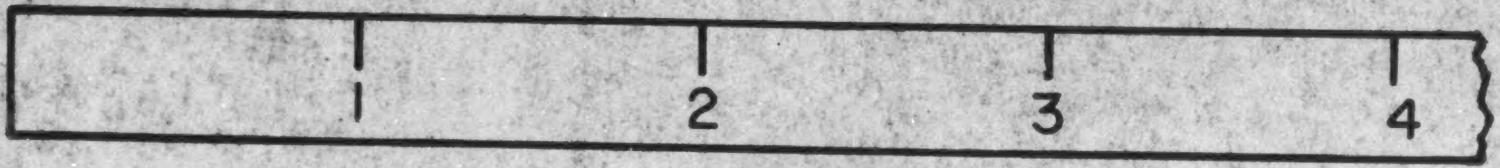
TO EACH POINT X ON $\overline{MN}$ THERE CORRESPONDS A POINT X' ON $\overline{RS}$



TO EACH POINT ON $\overline{RS}$ THERE CORRESPONDS A POINT Y' ON $\overline{MN}$

PRECISION - LINEAR MEASUREMENT

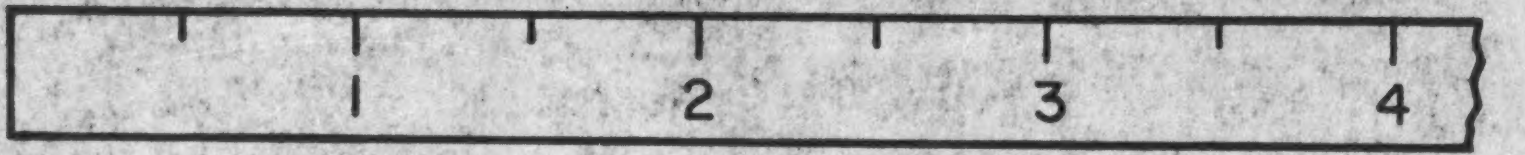




TO ONE-INCH PRECISION

AB IS 4," OR

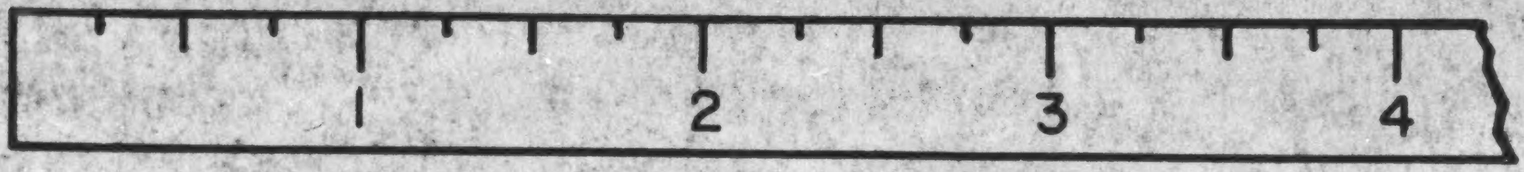
AB IS $(4\pm\frac{1}{2})''$



TO ONE-HALF INCH PRECISION

AB IS $3\frac{1}{2}$", OR

AB IS $(3\frac{1}{2} \pm \frac{1}{4})$"

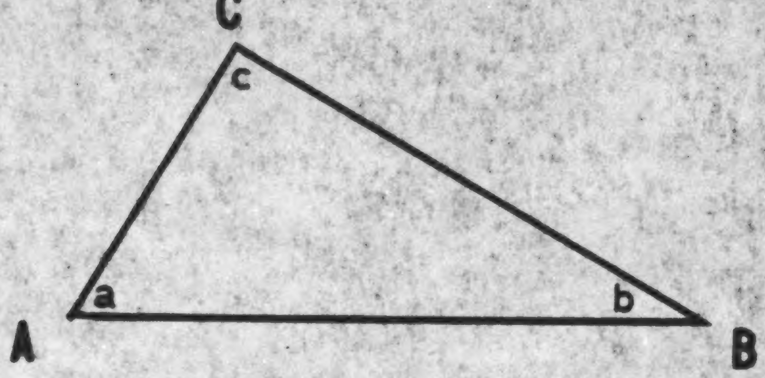


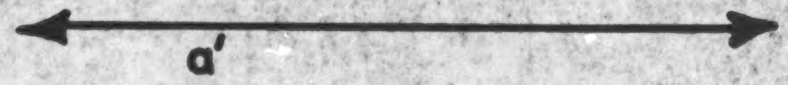
TO ONE-QUARTER INCH PRECISION

AB IS $3\frac{3}{4}$", OR

AB IS $(3\frac{3}{4} \pm \frac{1}{8})$"

ANGLES OF A TRIANGLE -- 180°





$m(\angle a) = m(\angle a')$

b'

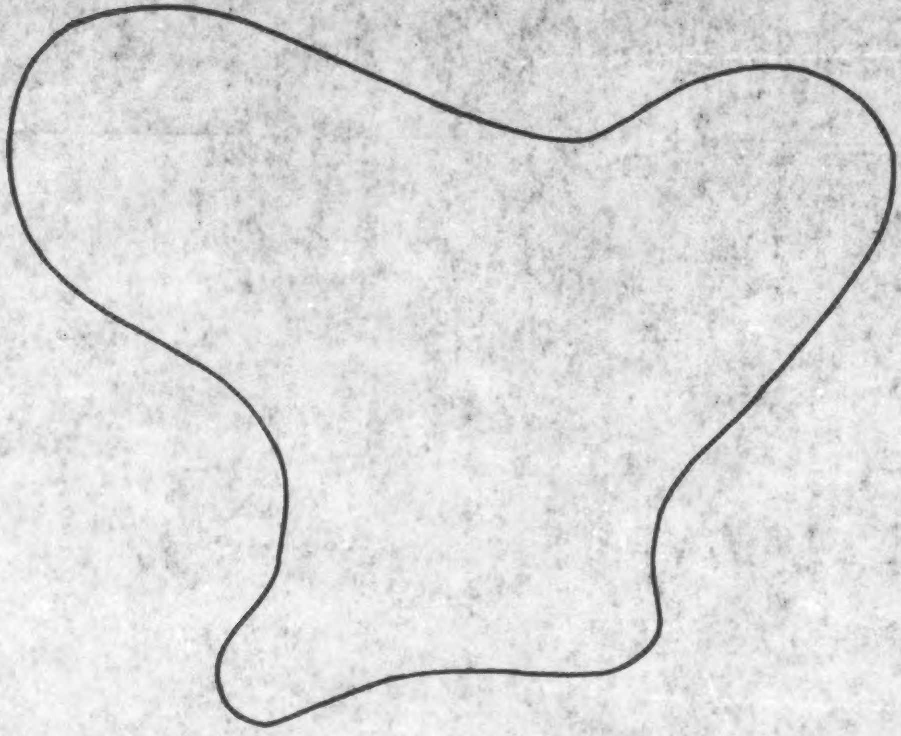
$, m(\angle b) = m(\angle b')$

12 - 3 Bottom

$m(\angle a') + m(\angle b') + m(\angle c) =$

$m(\angle a) + m(\angle b) + m'(\angle c) =$

AREA



13 - 1 Left

____ <A< ____

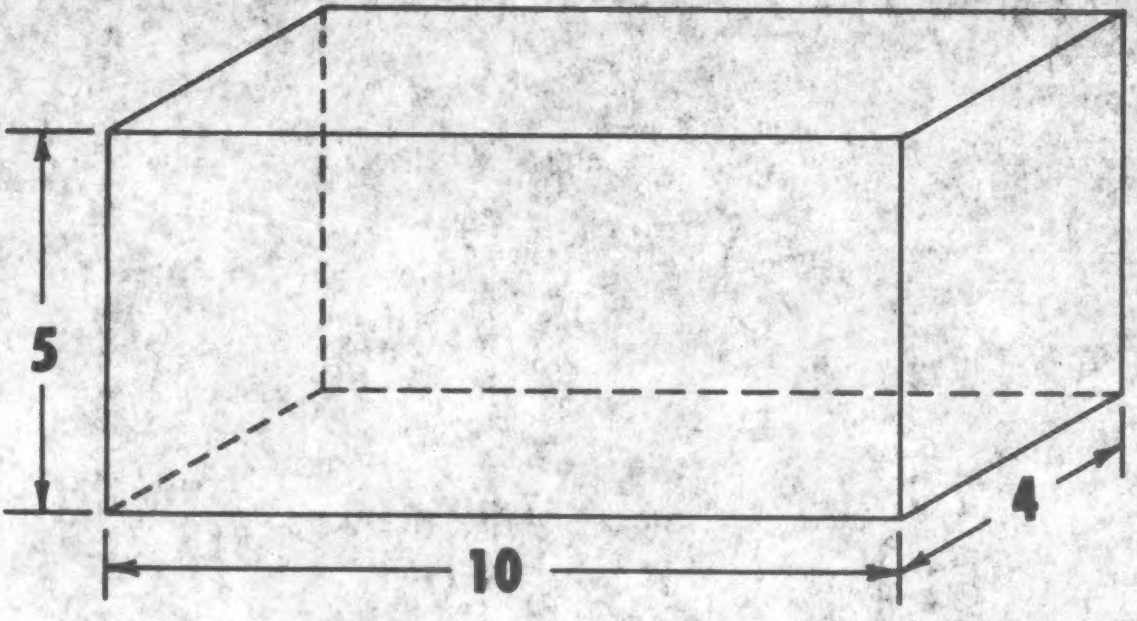
13 - 1 Right

<A<

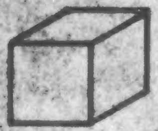
13 - 1 Bottom

<A<

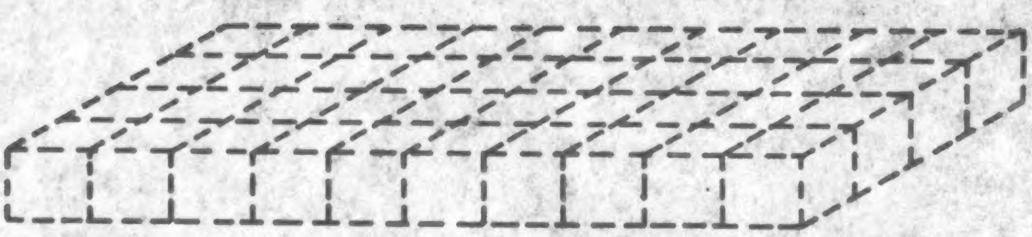
VOLUME



13 - 2 Left



1 cubic unit



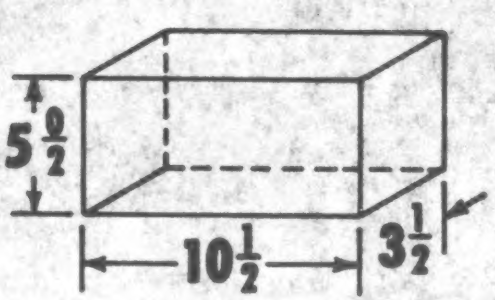
13 - 2 Right

$$B = (10 \times 4) = 40$$

$V = Bh = 40h = 40$ __________

$=$ __________

VOLUME – $\frac{1}{2}$ INCH PRECISION


$5\frac{0}{2}$
$10\frac{1}{2}$
$3\frac{1}{2}$

	MIN.	MEAS.	MAX.
LENGTH		$10\frac{1}{2}$	
WIDTH		$3\frac{1}{2}$	
HEIGHT		$5\frac{0}{2}$	
VOLUME		$\frac{11760}{64}$	
DIFF.		0	

13 - 3 Left

$10\frac{1}{4}$	$10\frac{3}{4}$
$3\frac{1}{4}$	$3\frac{3}{4}$
$4\frac{3}{4}$	$5\frac{1}{4}$
$\frac{10127}{64}$	$\frac{12900}{64}$
$\frac{1140}{64}$	$\frac{1633}{64}$

GREATEST POSSIBLE ERROR FOR VOLUME IS
CU. IN. TRUE VOLUME IS $183\frac{48}{64}$ $25\frac{33}{64}$ CU. IN.